The design of high power, high efficiency, multi-beam klystrons

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- Objectives: To find out how to choose the klystron output cavity parameters to get the highest possible efficiency under ideal conditions.
- A 6-disc model spreadsheet was developed using MathCAD. It optimises the parameters in the output cavity very quickly and gets results in seconds.
- The results from MAFIA and the 6-disc model are compared and efficiency errors are within 0.5dB using the same calculation conditions.
- The effects of perveance, gap voltage, initial bunch length and velocity distribution within bunch on efficiency are investigated using the 6-disc model.

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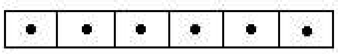
Introduction to the 6-disc model

• The bunch is represented by six rigid discs of charge whose **initial positions** are set at the output plane of the penultimate cavity by assuming a uniform distribution of charge within the bunch;



$$z_{init,i} = zstart + (3.5 - i) \times L$$

$$i = 1,2,...6$$

•All the charge from one electron wavelength λ_e of the un-modulated beam is assumed to have been concentrated into a length d (initial bunch length);

$$q = \pi \cdot b^2 \cdot \lambda_e \cdot \rho_0$$

$$\rho_0 = \frac{I_0}{\pi b^2 u_0}$$

$$\lambda_e = \frac{2\pi}{\beta_e}$$

$$\beta_e = \frac{\omega}{u_0}$$

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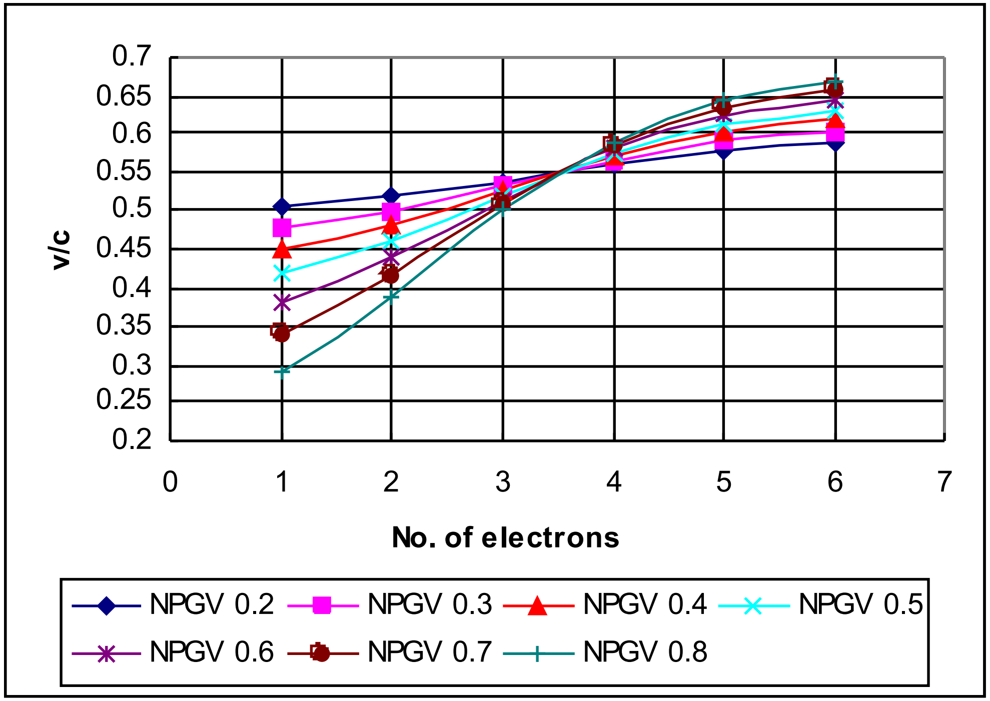
• The **initial velocities** are defined using the RF voltage across the penultimate gap so that the mean electron velocity is equal to the initial beam velocity but the bunch is tending to become compressed as it approaches the output gap.

$$u_{init,i} = c\sqrt{1-\frac{1}{\left[1+\left(\frac{eV_{init,i}}{m_0 c^2}\right)\right]^2}}$$

$$V_{init,i} = V_0\left[1-\gamma_p \cdot \sin\left(\beta_e L\left(3.5-i\right)\right)\right]$$

$$\gamma_p = \frac{V_p}{V_0}$$

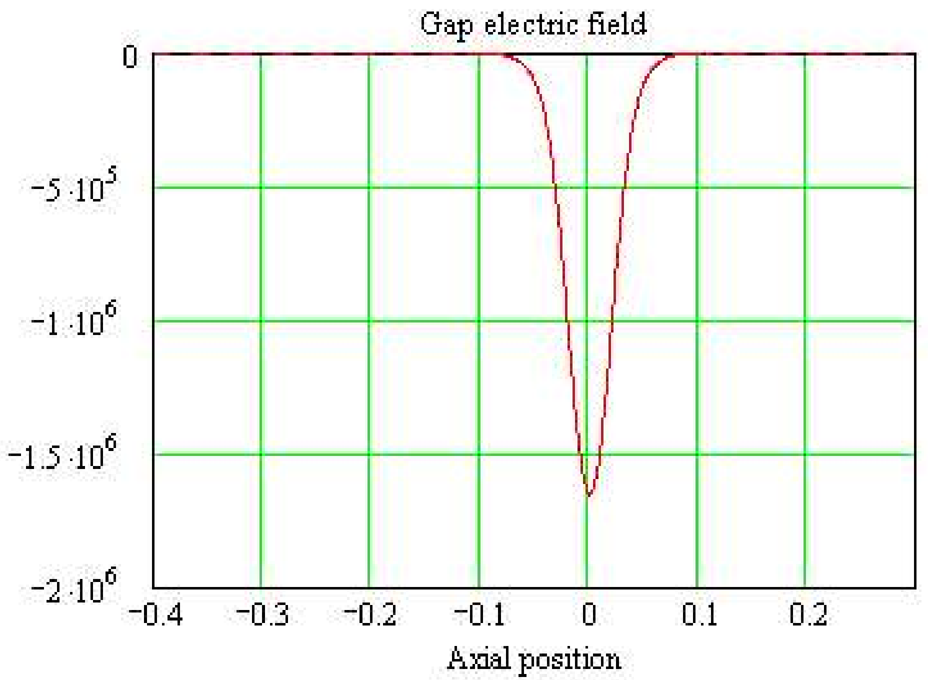
Normalised penultimate cavity voltage



• The **interaction field** on the axis is related to the field in the gap by an integral involving the I_0 Bessel's function:

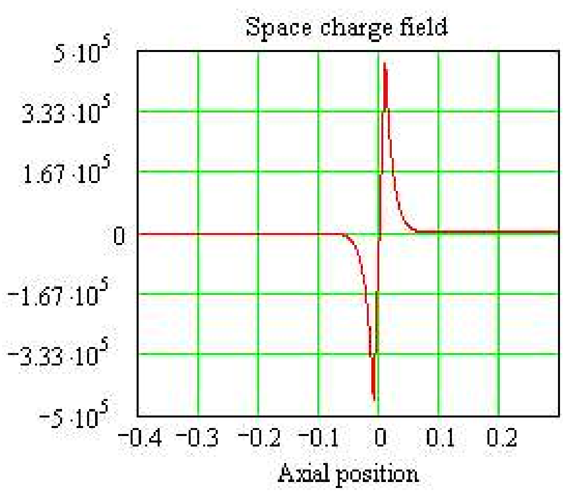
$$E(z) = \frac{E_0}{2\pi} \int_{-2\pi}^{2\pi} \frac{1}{I_0(\theta \cdot \frac{a}{g})} \cdot \frac{\sin(0.5 \cdot \theta)}{0.5 \cdot \theta} \cdot e^{-j\theta \cdot \frac{z}{g}} d\theta$$

$$E_0 = \frac{V_g}{g}$$

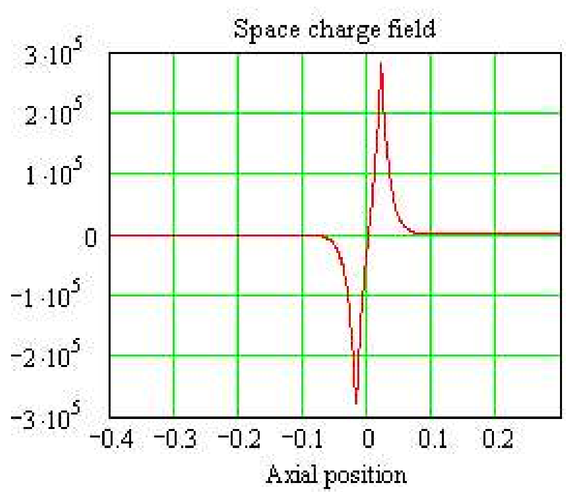


• Reference [1] gives an expression for the **space charge field** (omitting the the field of adjacent bunches) as:

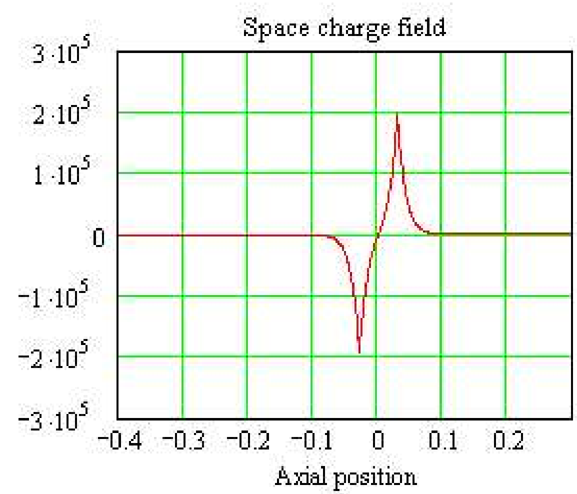
$$Es(z,L) = \begin{cases} \dfrac{2\cdot a\cdot b\cdot \rho_0\cdot \lambda_e}{3\cdot \varepsilon_0\cdot L}\cdot\dfrac{z}{|z|}\cdot\displaystyle\sum_{m=1}^{10}\dfrac{\mu_m}{(\mu_m\cdot a)^3}\cdot\left[\dfrac{J1(\mu_m\cdot b)}{(J1(\mu_m\cdot a))^2}\right]\cdot\sinh\left(\dfrac{\mu_m\cdot L}{2}\right)\cdot e^{-\mu_m\cdot|z|} & if\ |z| \ge 0.5\cdot L \\ \dfrac{2\cdot a\cdot b\cdot \rho_0\cdot \lambda_e}{3\cdot \varepsilon_0\cdot L}\cdot\displaystyle\sum_{m=1}^{10}\dfrac{\mu_m}{(\mu_m\cdot a)^3}\left[\dfrac{J1(\mu_m\cdot b)}{(J1(\mu_m\cdot a))^2}\right]\cdot\sinh(\mu_m\cdot z)\cdot e^{-\frac{\mu_m\cdot L}{2}} & if\ |z| < 0.5\cdot L \end{cases}$$



Bunch length $0.25\lambda_e$



Bunch length $0.5\lambda_e$



Bunch length $0.75\lambda_e$

[1] J. Richard Hechtel, The effect of potential beam energy on the performance of linear beam devices, IEEE Trans. on ED, Vol ED-17, No. 11, November 1970

Efficiency calculation

- Motion equations are solved using Runge-Kutta method;
  - Electric field acting on electrons:

$$E_i = E(z_i)\cos(\omega \cdot t + \phi) + \sum_{j=1}^{6} Es(z_i - z_j, L)$$

  - Motion equations:

$$\frac{d^2 z_i}{dt^2} = \frac{E_i e}{m}$$

- The initial and final kinetic energies are estimated from the energies of the 6 sample electrons;

$$KE_{initial} = \left(\frac{q \cdot c^2}{6\eta}\right) \cdot \sum_{i=1}^{6} \left[\left(\frac{1}{\sqrt{1 - \left(\frac{v_{initial,i}}{c}\right)^2}}\right) - 1\right]$$

- RF power and efficiency:

$$P_{rf} = \left[KE_{initial} - KE_{final}\right] \cdot f$$

$$\eta = \frac{P_{rf}}{P_{dc}} \times 100\ \%$$

Klystron output cavity investigation using MAFIA

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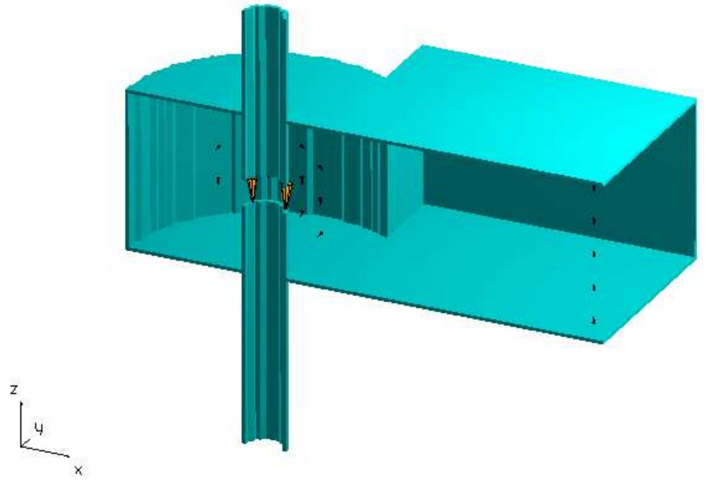

$$f = 348.85MHz \qquad \frac{R}{Q} = 122\Omega$$

Re-entrant cavity with a waveguide coupling

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MAFIA simulation

- Bunch length is half of the electron wavelength;
- No electrons between bunches;
- Velocities vary linearly within bunches;

- The cavity is driven only by the incoming bunches;
- Loaded shunt impedance of the cavity is adjusted by changing the coupling between the cavity and the waveguide;
- Cavity tuning is realised by changing the bunch repetition rate;

MAFIA simulation

- Two methods to calculate the power to the waveguide:

  1) Poynting Vector was used to calculate the RF power flow in the output waveguide;

  2) Gamma of 9 representative particles was used to calculate the kinetic energy loss of one bunch.

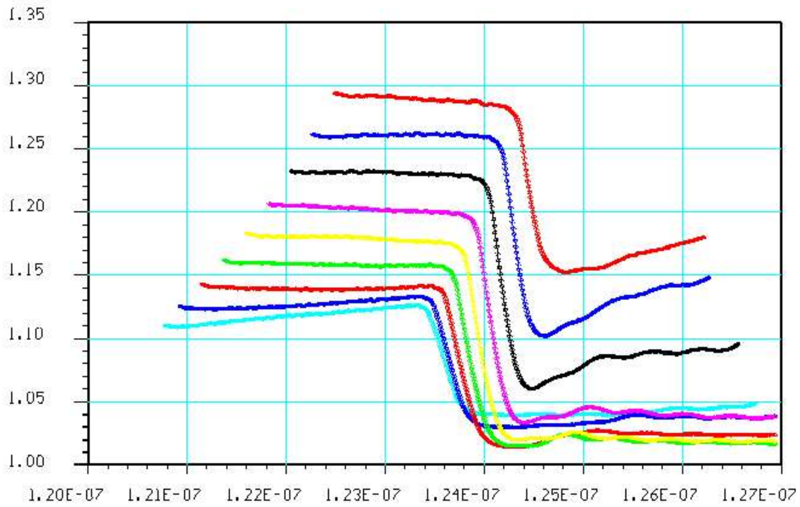
$$Poynting = \frac{B_y \cdot E_z}{4\mu_0}$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

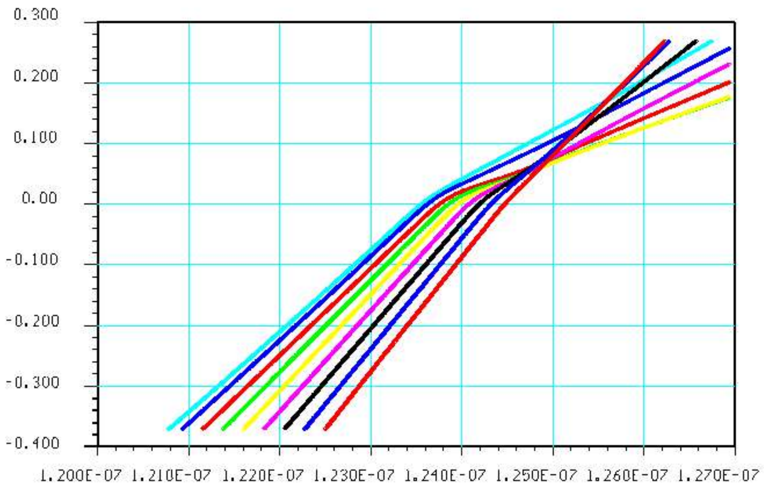
$$KE = mc^2\left(\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}-1\right) = mc^2(\gamma - 1)$$

By and Ez are the magnetic and electric field amplitude at the centre of the waveguide end plane, respectively.

Gamma and axial position variation with time from MAFIA

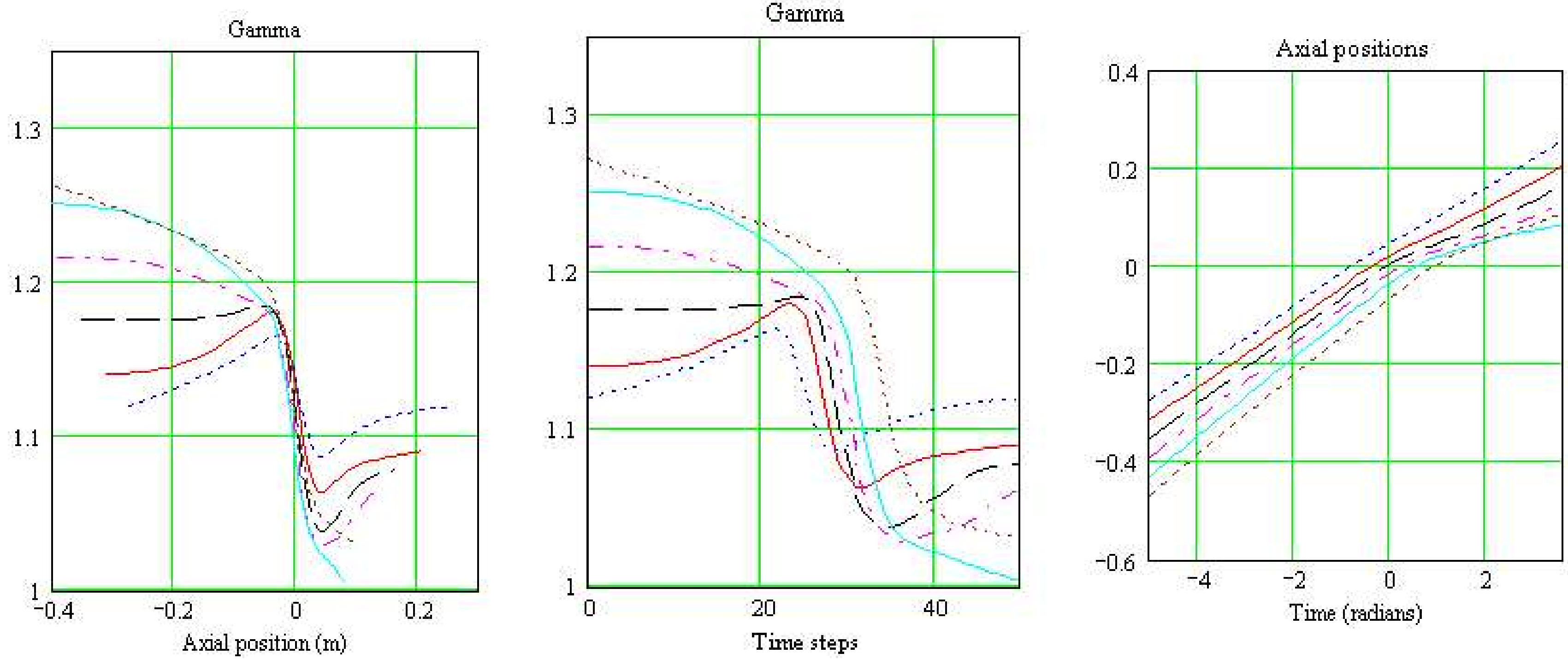


Gamma

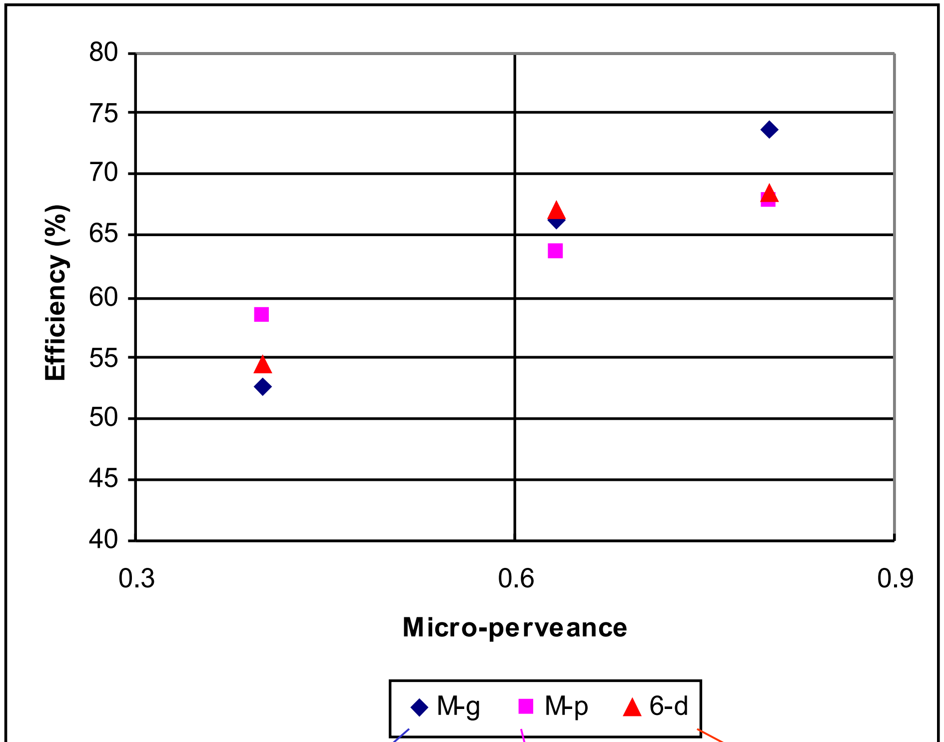


Axial position

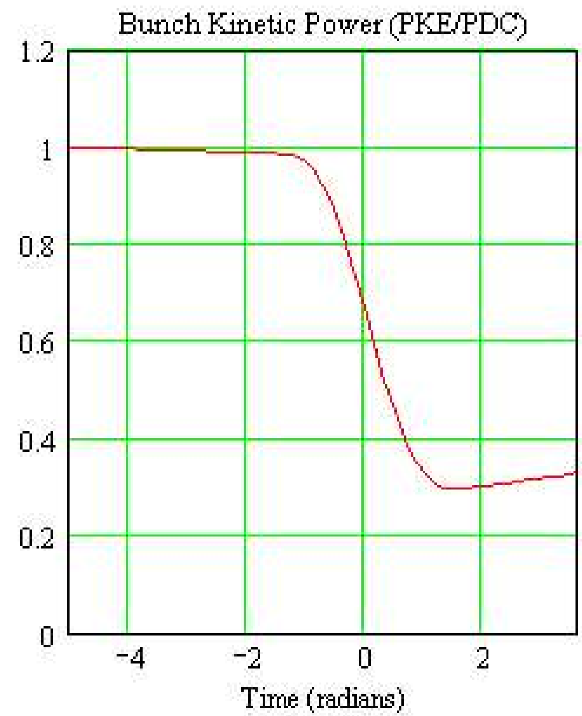
Gamma and axial position variation with time from the 6-disc model



Efficiencies



$$KE_{initial} = \left(\frac{q \cdot c^2}{6\eta}\right) \cdot \sum_{i=1}^{6} \left[\left(\frac{1}{\sqrt{1 - \left(\frac{v_{initial,i}}{c}\right)^2}} \right) - 1 \right]$$



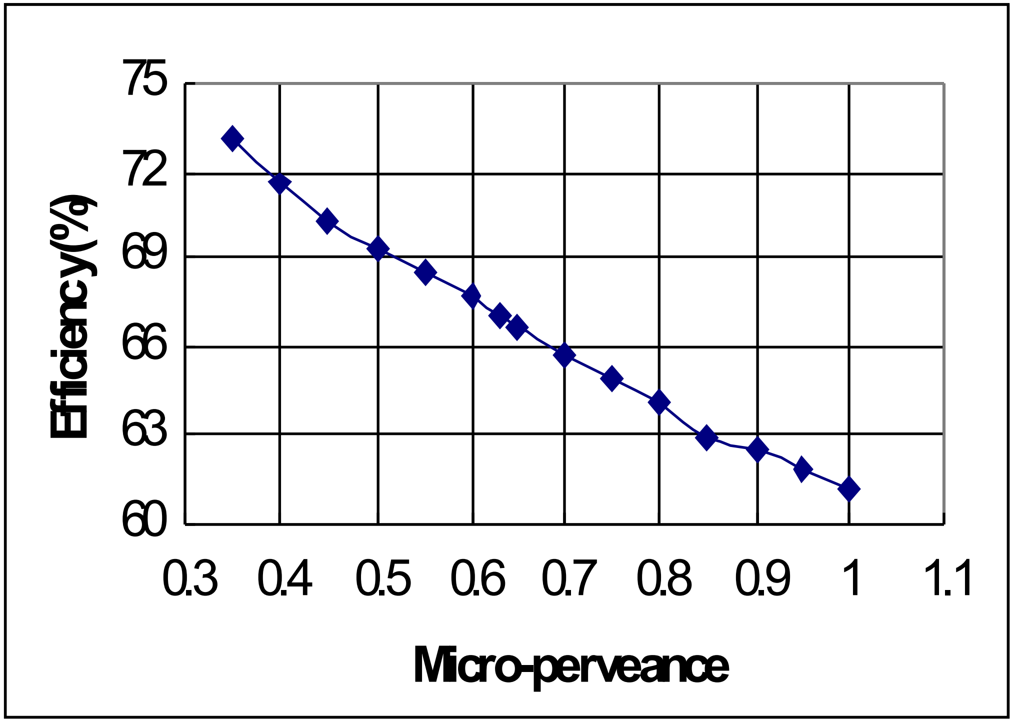
MAFIA gamma calculation

MAFIA Poynting vector calculation

6-disc model

The errors are within 0.5dB.

Efficiency vs perveance



Linear fit of the efficiency~perveance curve

$$\eta = 0.7853 - 0.1788 P_{\mu}$$

Dependence of efficiency on perveance

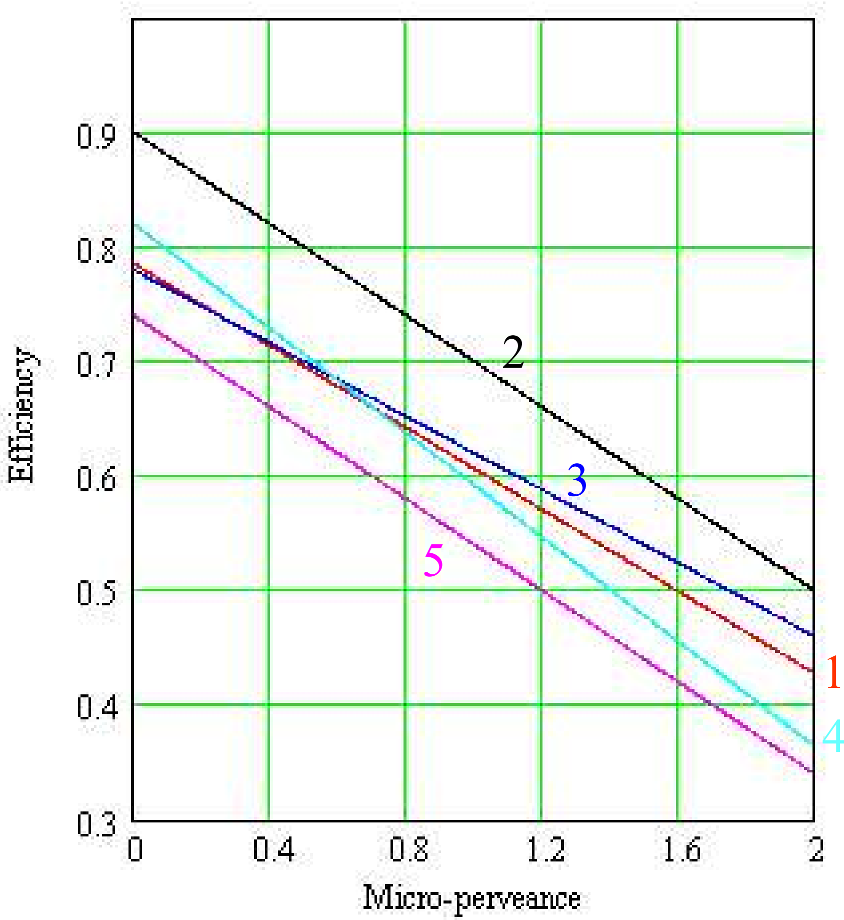
1. 6-disc $\eta = 0.7853 - 0.1788 P_{\mu}$

2. Theoretically* $\eta = 0.90 - 0.20 P_{\mu}$

3.Thales $\eta = 0.78 - 0.16 P_{\mu}$

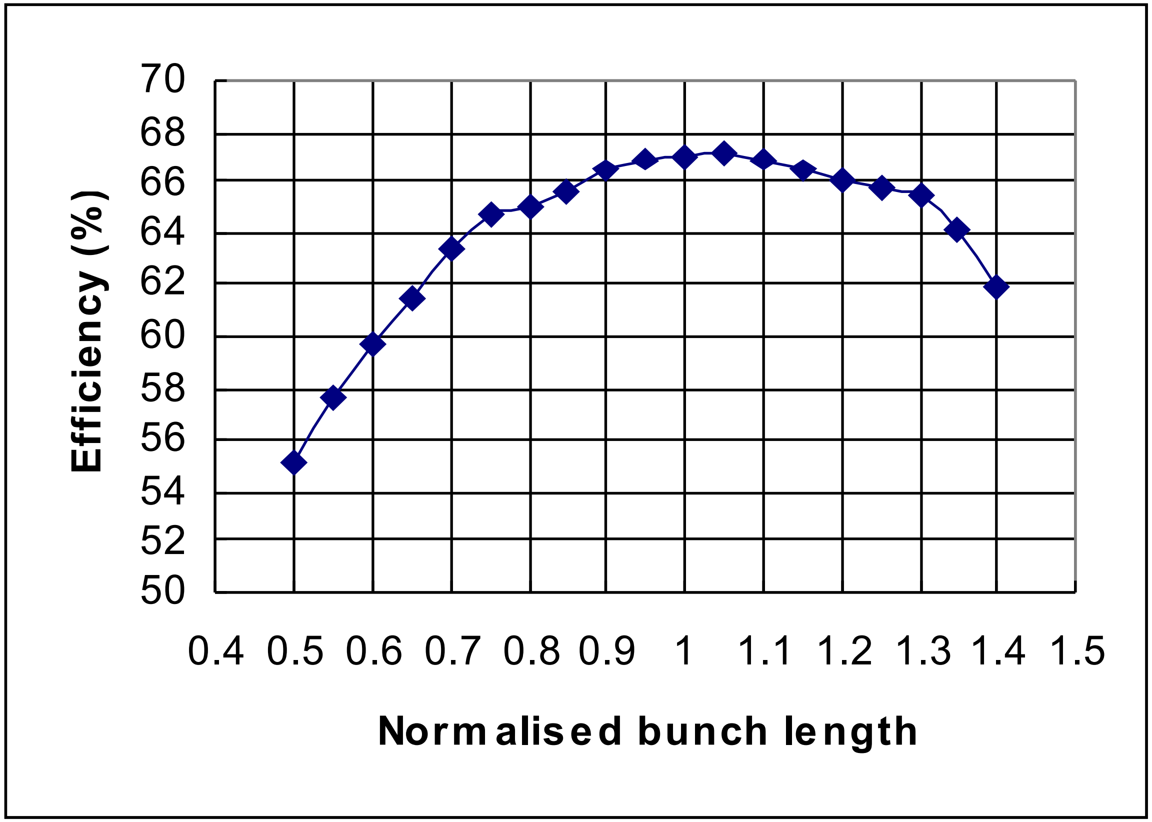
4.LANL $\eta = 0.82 - 0.228 P_{\mu}$

5.SLAC $\eta = 0.74 - 0.20 P_{\mu}$



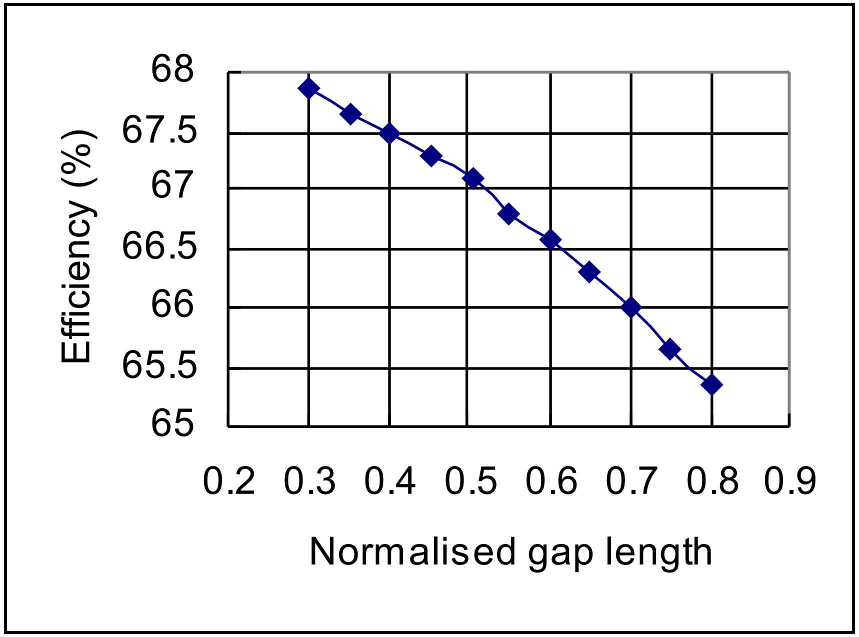
* by Robert S. Symons

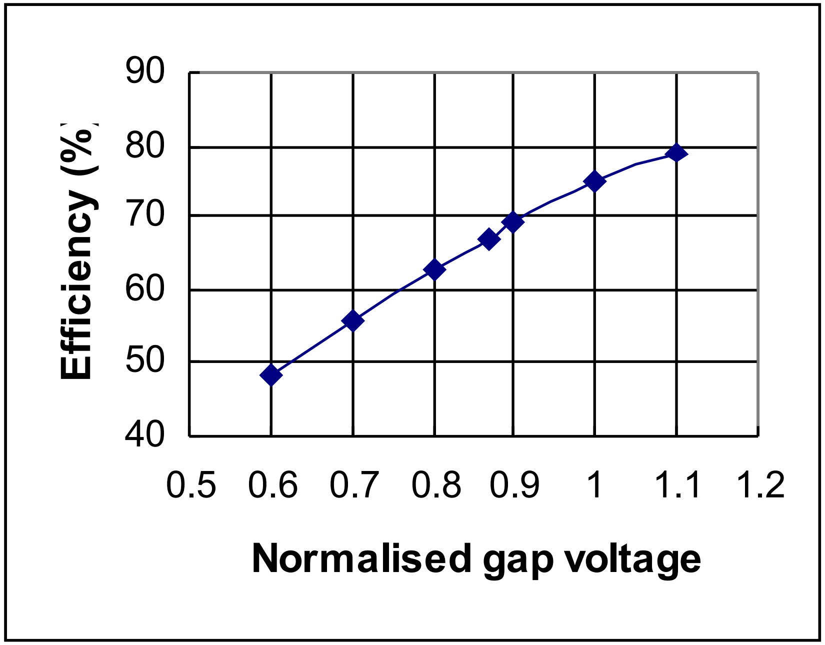
Dependence of efficiency on initial bunch length



Normalised Bunch Length is defend as: $NBL = \frac{\beta_e d}{\pi}$

Dependence of efficiency on output gap length and gap voltage





$$NGL = \beta_e \times g$$

$$NGV = \frac{V_g}{V_0}$$

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Conclusion

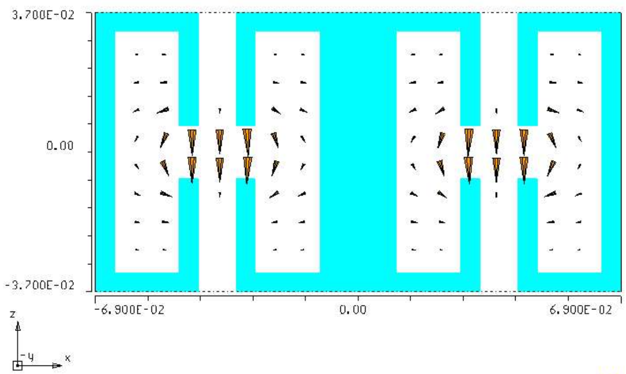
- A simple six-disc model was developed for parameter optimisation of the klystron output cavity and results can be obtained very quickly;
- The results from the 6-disc model were verified by comparison with MAFIA simulation.The efficiency from the model and from MAFIA agree to within 0.5dB.
- Model has been used to investigate how the efficiency of a klystron varies with perveance, gap length, gap voltage, gap length and bunch length.

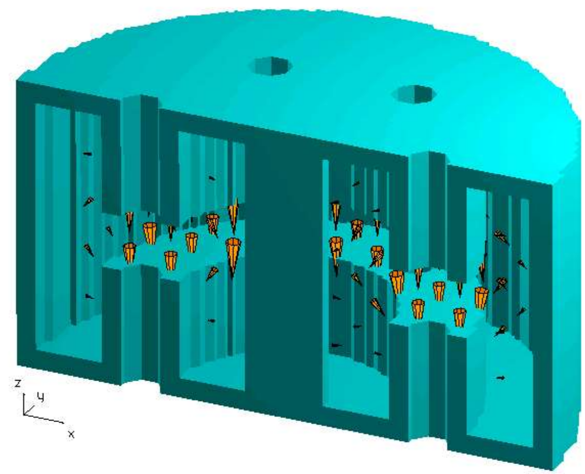
Work on coaxial output cavity:

- **Objectives: To prove that the coaxial cavity is possible with reasonable efficiency using coaxial output structure;**
- **Preliminary design obtained from the coaxial cavity design spreadsheet;**
- **Output cavity simulation in 6-disc model shows an efficiency of 66%;**
- **Impedance transforming technology used to give a good coupling between the output cavity and the loading;**
- **MAFIA simulation shows an efficiency of 58%;**
- **Simulation results shows losing beams will not affect the uniformity of the electric field along the ridge, just weaken the field and make the efficiency go down.**

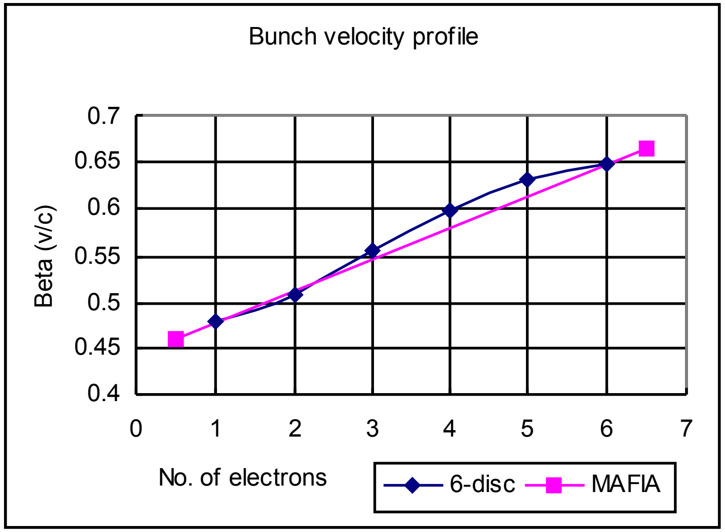
Cavity simulation

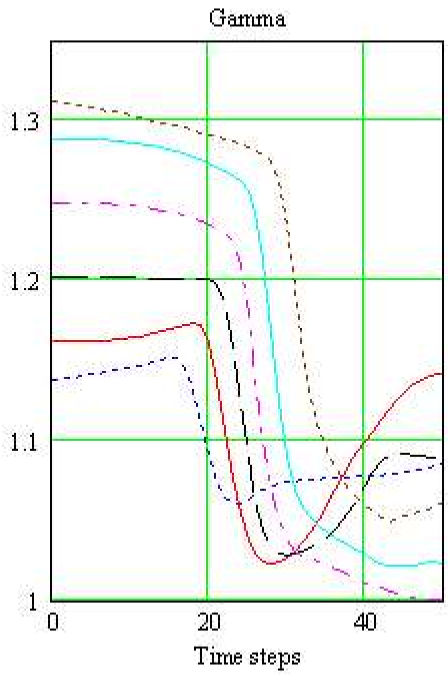
	Frequency (GHz)	Characteristic impedance (Ohm)
MWS	1.28766	
MAFIA	1.30006	25.8
Spreadsheet	1.30819	24.8

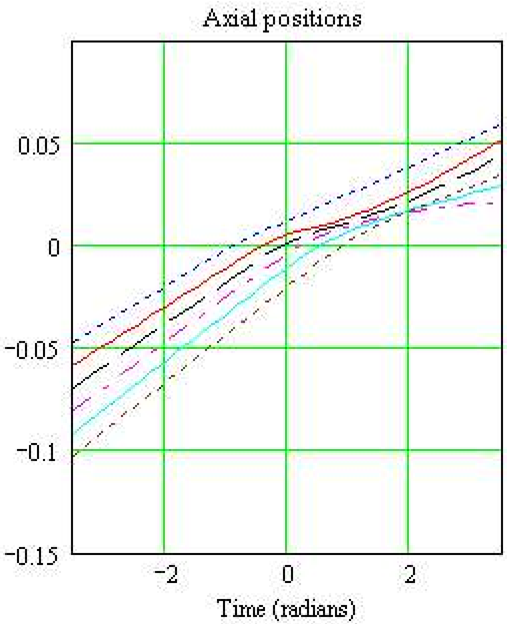


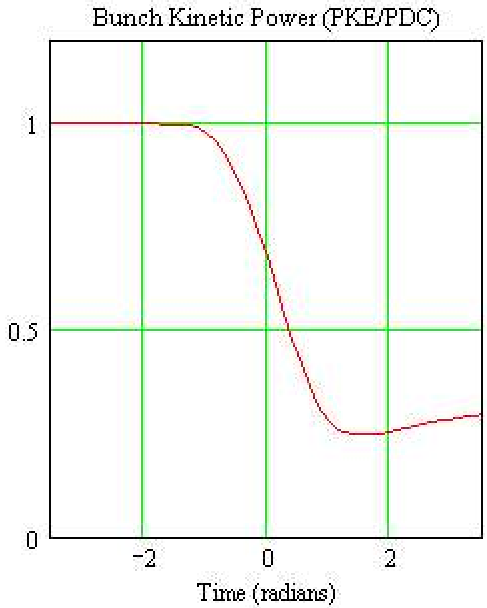


Six-disc model simulation









66% efficiency

Cavity impedance matching

Circuit analysis shows that the impedance of the cavity is less than 0.1 Ohm, two quarter wave impedance transformers are used to get a good matching between the cavity and the load to avoid over-coupling or less-coupling.

Impedance of coaxial line:

$$Z = \frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}}\ln\left(\frac{R}{r}\right) = 60\ln\left(\frac{R}{r}\right)$$

	Transformer 1	Transformer 2
Impedance (Ω)	30	59.6

After the impedance transformer, the impedance is around 0.35Ohm.

MAFIA simulation

- Two methods are used to calculate the power flow in the waveguide:

  1) Area-integral of Poynting Vector at the end plane of the coaxial output waveguide using a waveguide boundary condition;

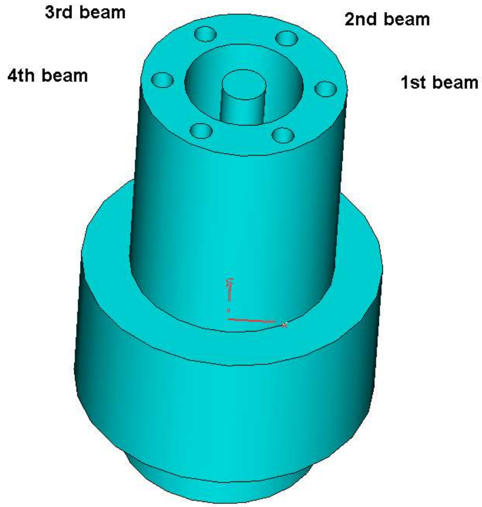
  2) Gamma calculation of 9 representative particles in one bunch were used to calculate the kinetic energy loss of one bunch in one RF cycle;

- Losing beams (input bunches are the same, not considering the bunching section)

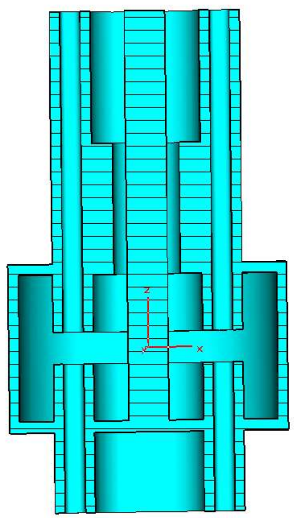
  1) Losing beams will not affect the uniformity of the electric field around that gap ridge;

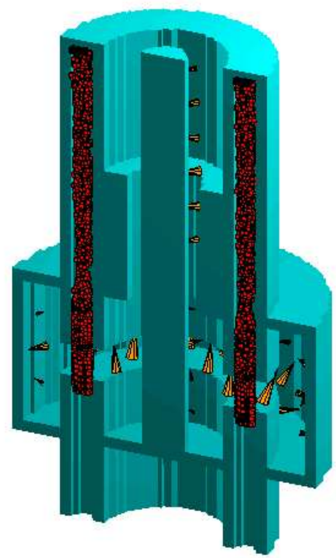
  2) The weakened electric field will lower down the efficiency and the output power;

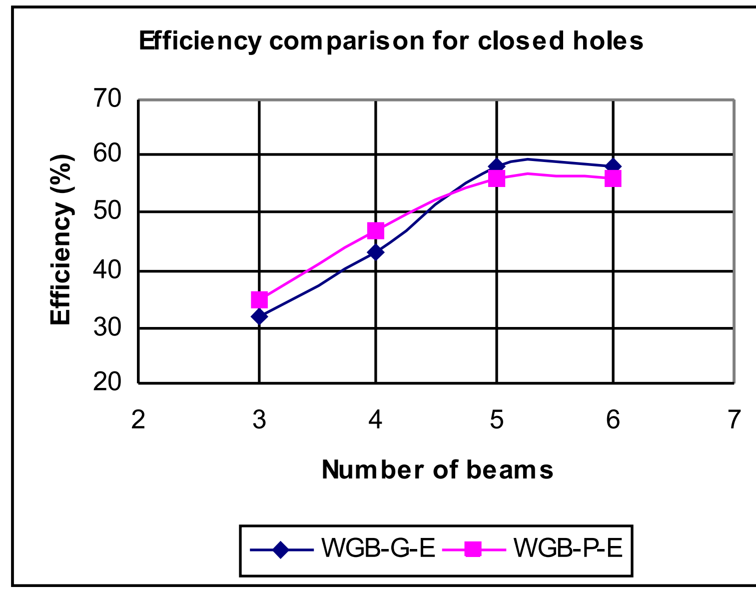
MAFIA simulation



Holes are closed at the end in the simulation.

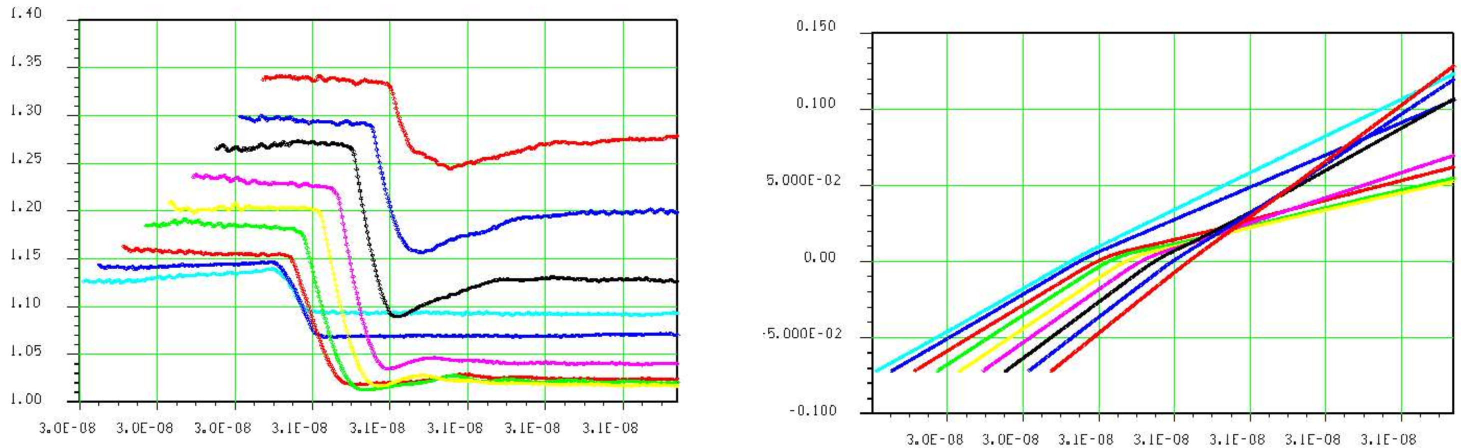




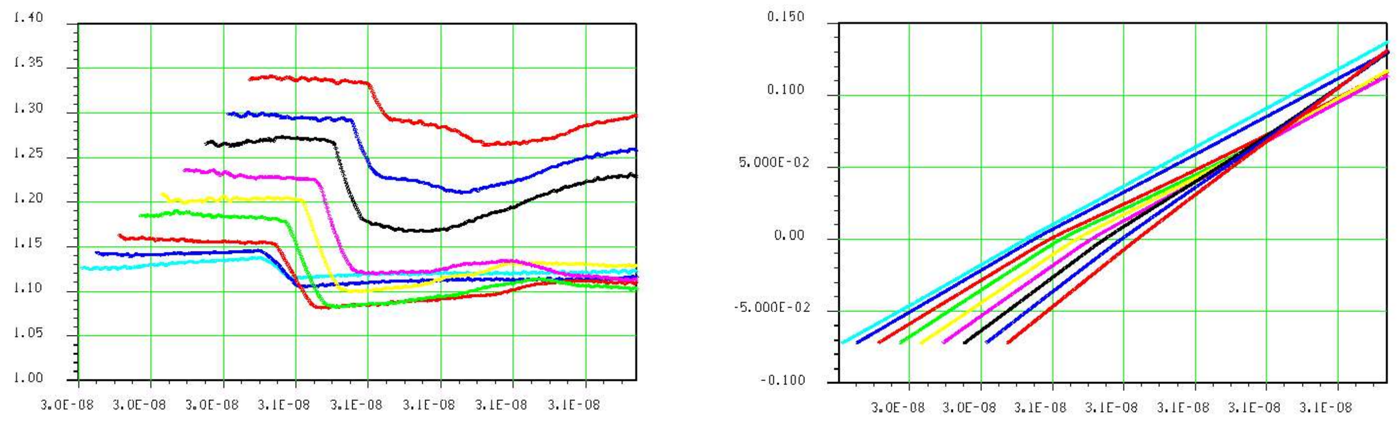


Beam numbers	3	4	5	6
Efficiency from Gamma (%)	32	43	58	58
Efficiency from pointing Integral (%) (and power)	35 (2.5/7.2MW)	47 (4.5/9.6MW)	56 (6.7/12MW)	56 (8/14.4MW)

MAFIA simulation results



Gamma and axial position variation of the representative particles with all 6 beams



Gamma and axial position variation of the representative particles with losing 3 beams

Beam bunching investigation

Model for the beaming bunching investigation

•The interaction field for unit gap voltage on the axis of a gap of length at position is found from the Fourier transform of the field in the gap

$$E(z) = \frac{1}{2\pi g} \int_{-2\pi}^{2\pi} \frac{1}{I_0(\theta \cdot \frac{a}{g})} \cdot \frac{\sin(0.5 \cdot \theta)}{0.5 \cdot \theta} \cdot e^{-j\theta \cdot \frac{z - zg}{g}} d\theta$$

$$E(z,t) = \sum_{i=1}^{NG} Eg(g_i, zg_i, z) \cdot Vg_i \cdot \cos[2\pi (f_i + DF_i) \cdot t + \phi_i]$$

•Space charge force: integral of the RF space charge modified by the square of the plasma frequency reduction factor R.

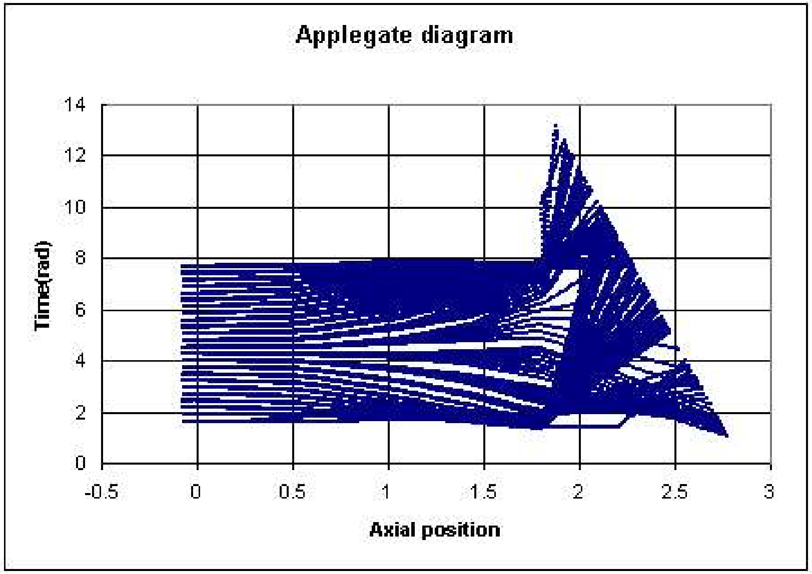
$$\frac{d^2 z}{dt^2} = \eta \left[1 - \frac{v^2}{c^2}\right]^{3/2} \cdot \left[E(z_i, t) + \left(\frac{F^2 \rho_0}{\varepsilon_0}\right) \cdot \left[\frac{x_i \cdot \lambda_e}{24} - z_i + 0.5(z_6 + z_7) \right] \right]$$

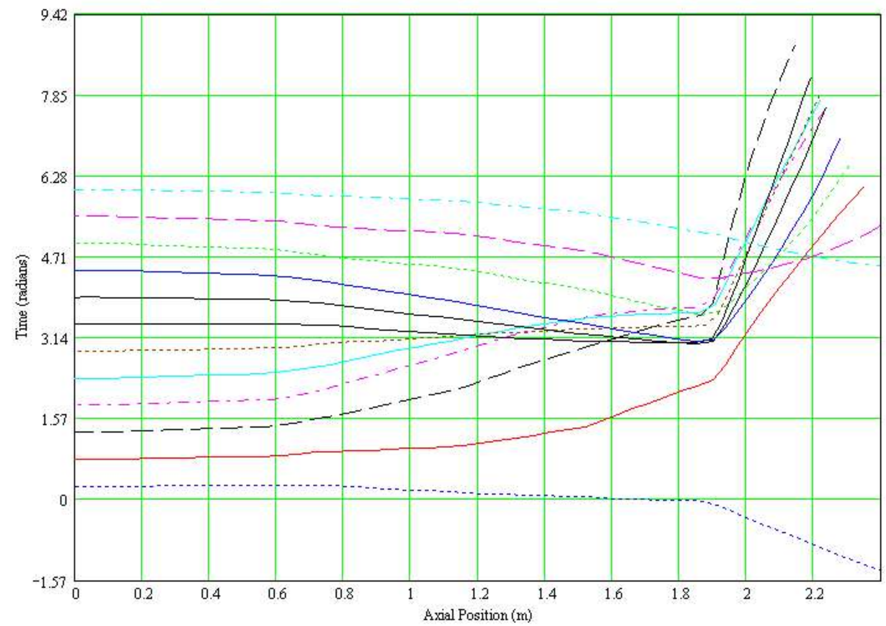
•The motion equations are solved by using a Runge-Kutta method.

•The gap voltage, gap phase, frequency of each cavity can be varied to see the bunching dynamics.

Applegate diagram from Ring model and the Spreadsheet (50W input power, 220Gauss)

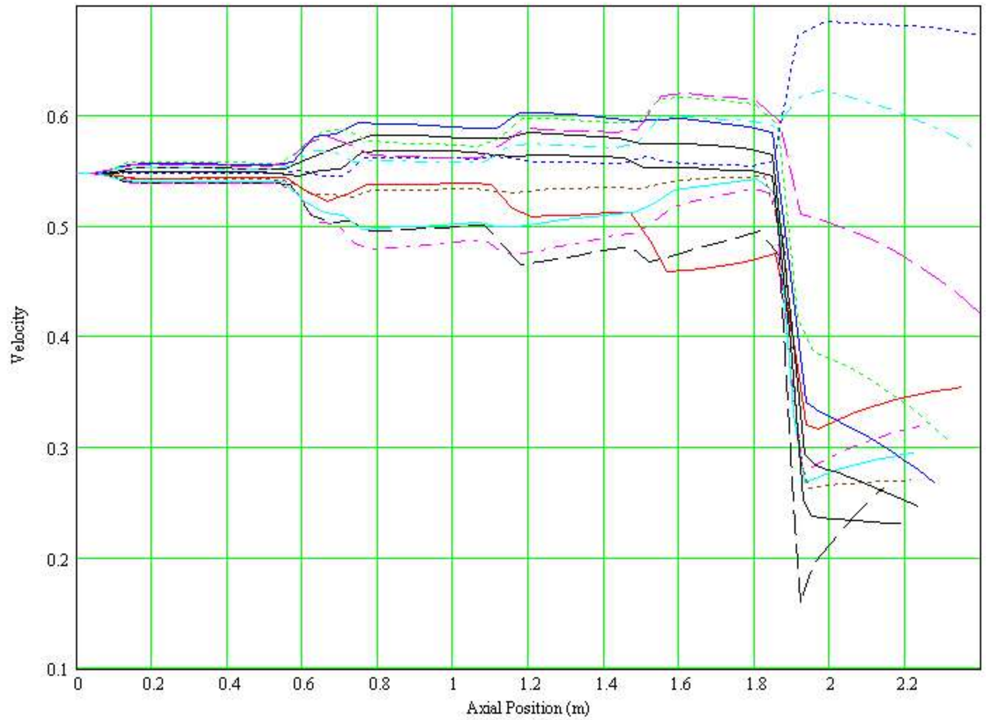
Time(rad): $\omega \cdot (t - \frac{z}{u_0})$

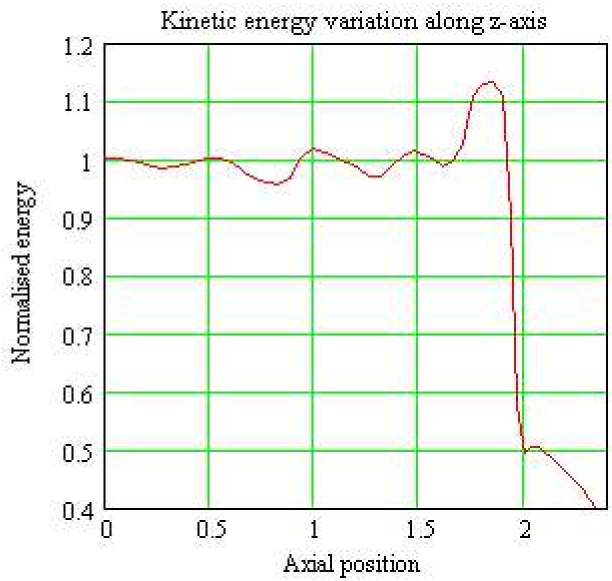




Gap number	1	2	3	4	5	6
Gap voltage (KV)	4.81	17.52	11.21	17.80	24.26	92.12
Initial gap field phase (rad)	0	-0.3770	4.3959	-1.2245	-0.1034	-0.4695
Delta Frequency (MHz)	0.0	0.0	352.21	0.0	0.0	0.0

Data from ring model (50W input power, 220Gauss)



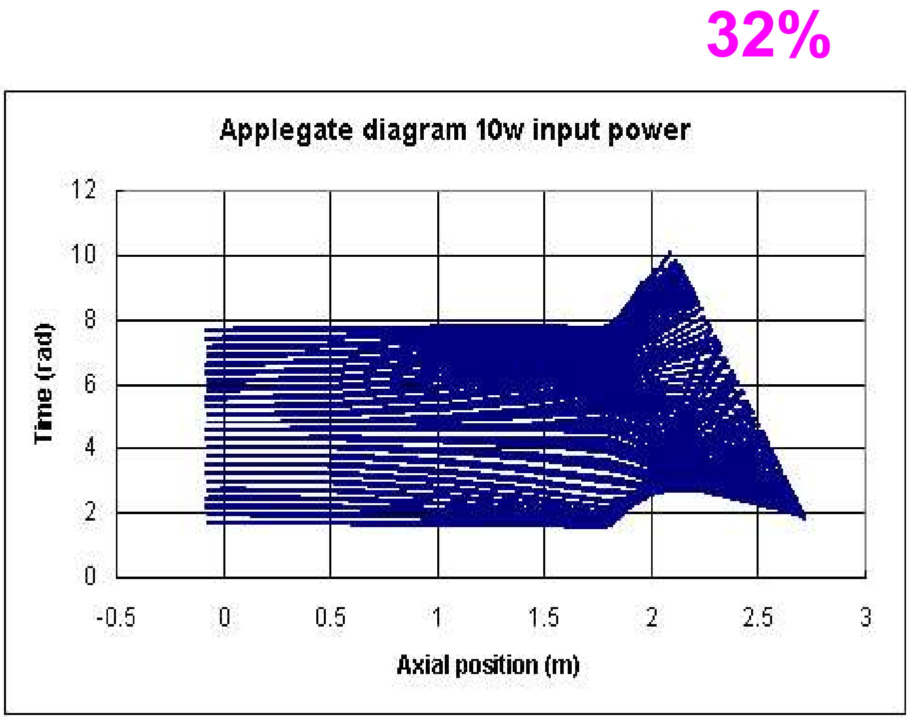


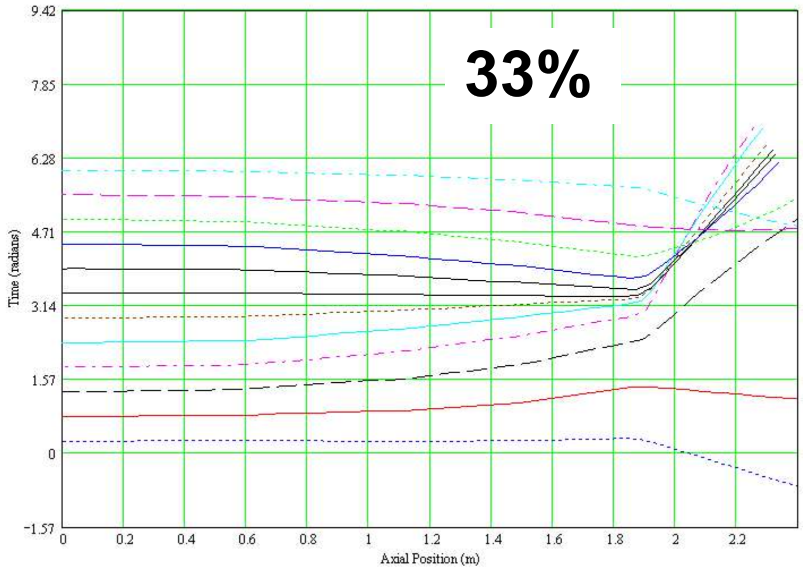
Efficiency:

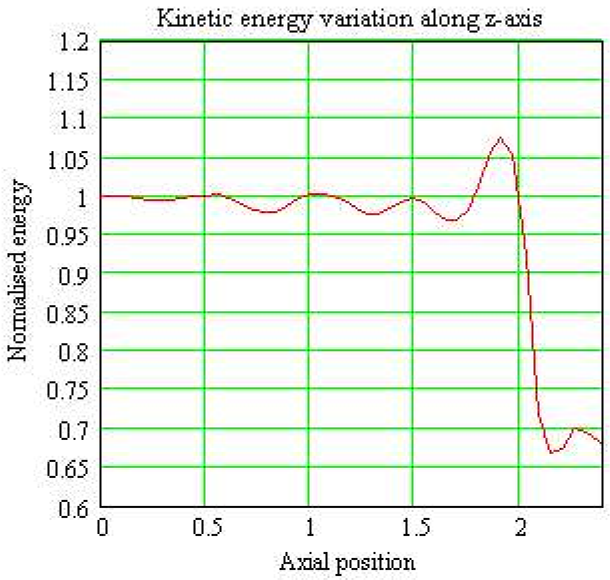
55% (220Gauss) from ring model, beam interception is up to 34%;

50% from the Spreadsheet.

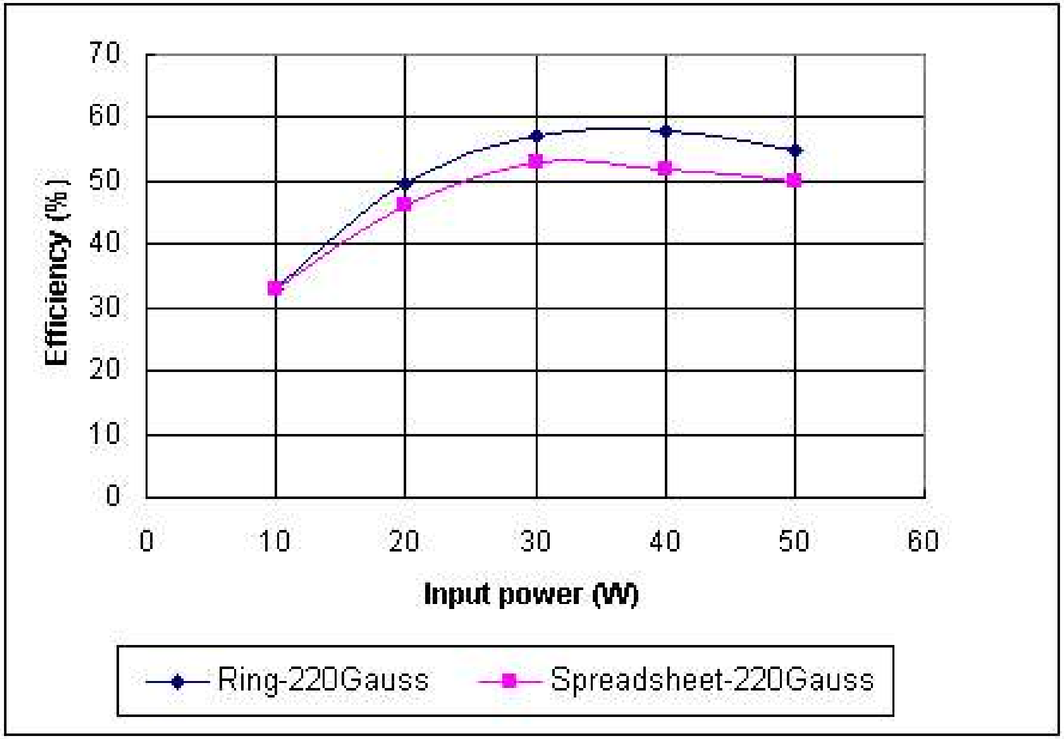
10W input power





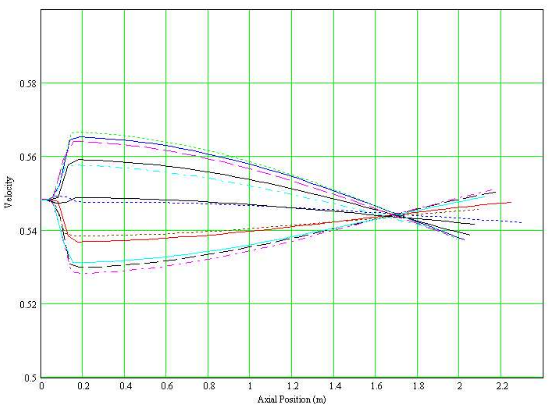


Results comparison for 220Gauss

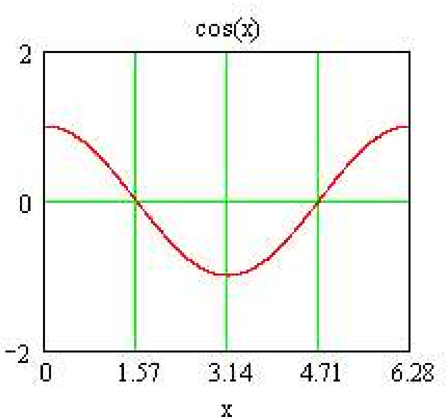


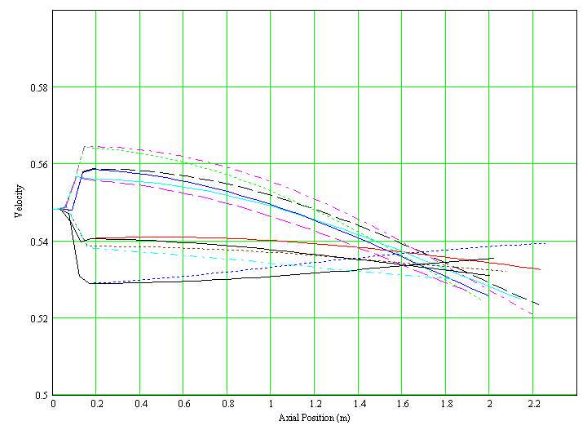
Injection of harmonics into the input cavity

Trying to get ideal bunch for interaction with RF gap field, input cavity is injected with a harmonic , or together with fundamental frequency.

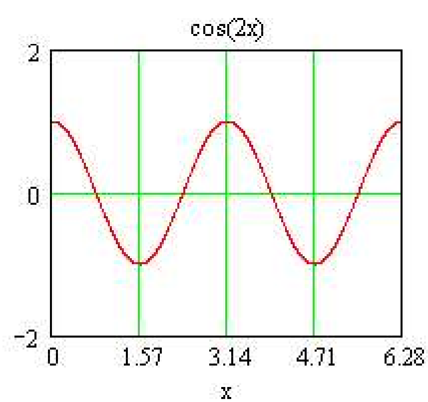


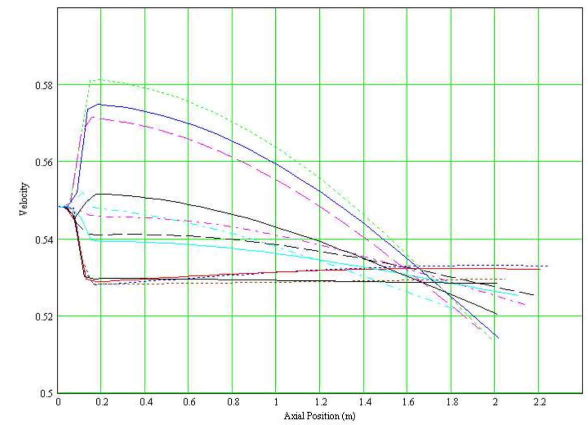
Fundamental



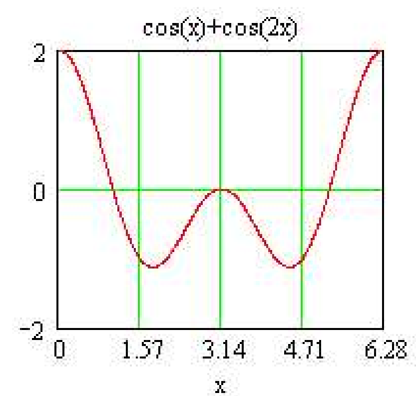


Second harmonic

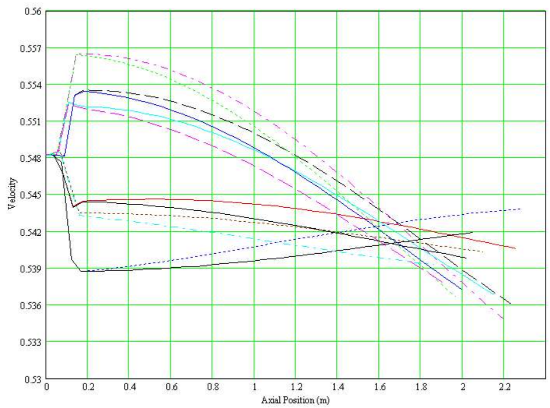




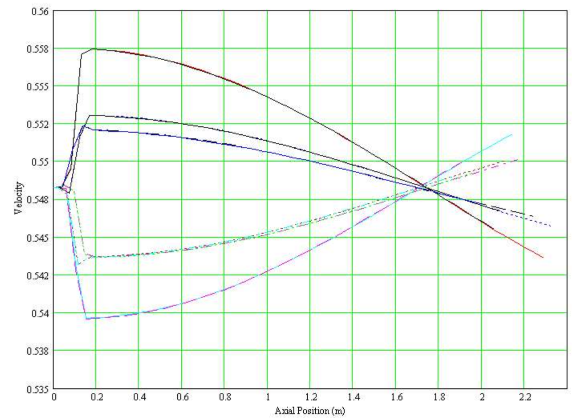
Both



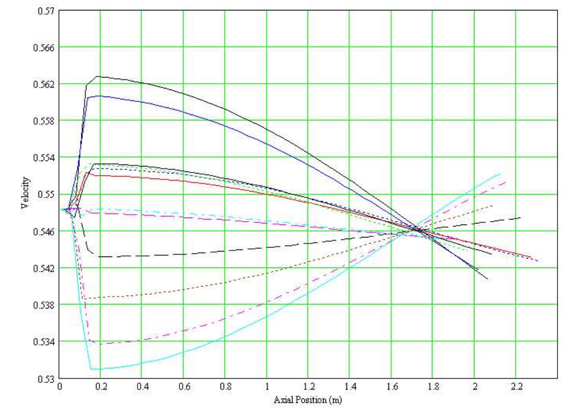
The asymmetry can be compensated by adjusting the phase of the harmonic.



0 phase, 2^{nd} harmonic

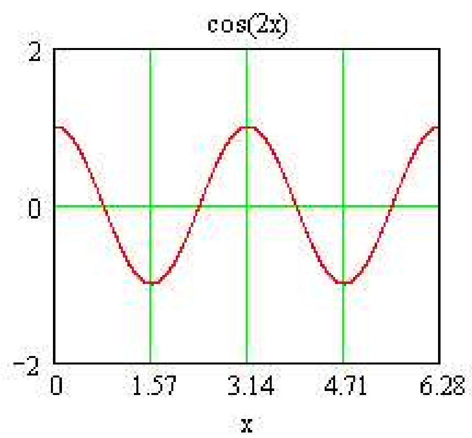


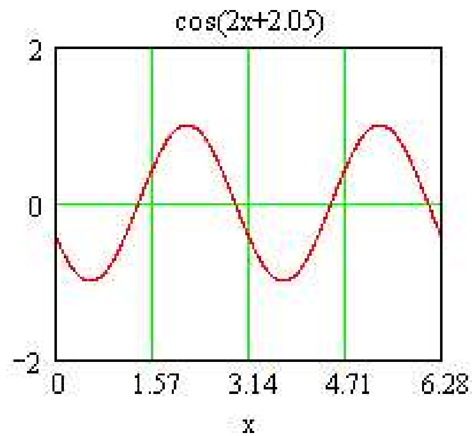
2.05phase, 2^{nd} harmonic

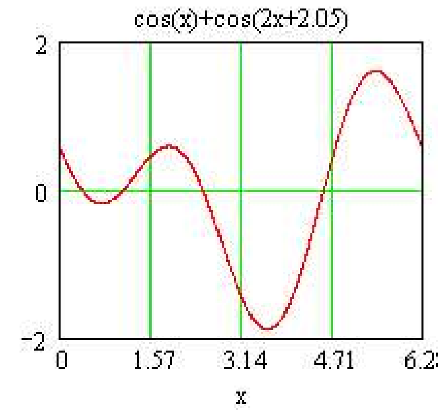


2.05 phase, 2^{nd} harmonic

0 phase, fundamental



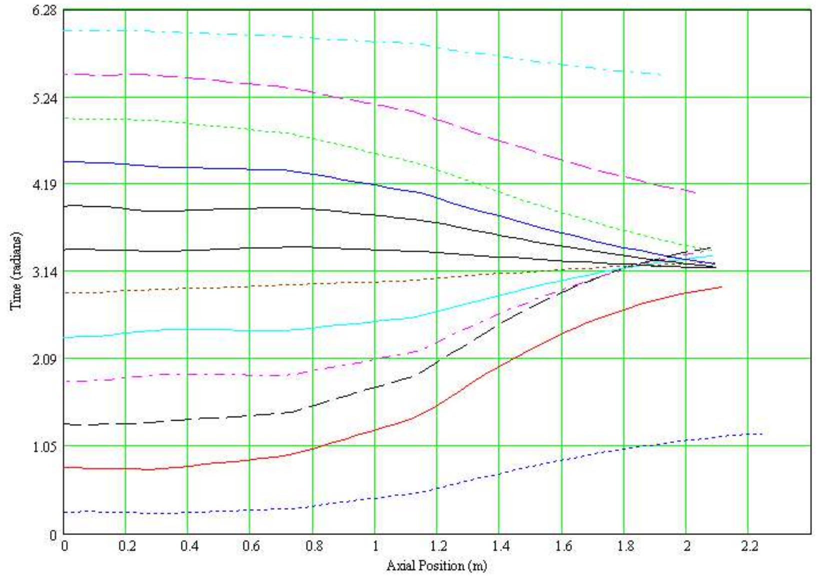


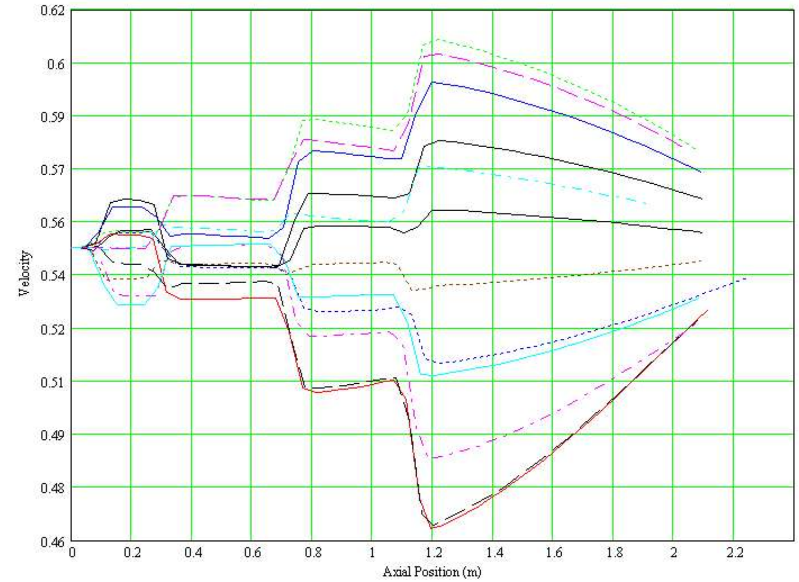


Quarter plasma wavelength from the plots: 1.6m, calculation results: $\frac{\lambda_q}{4} = 1.625m$

Bunching using harmonic in the input cavity

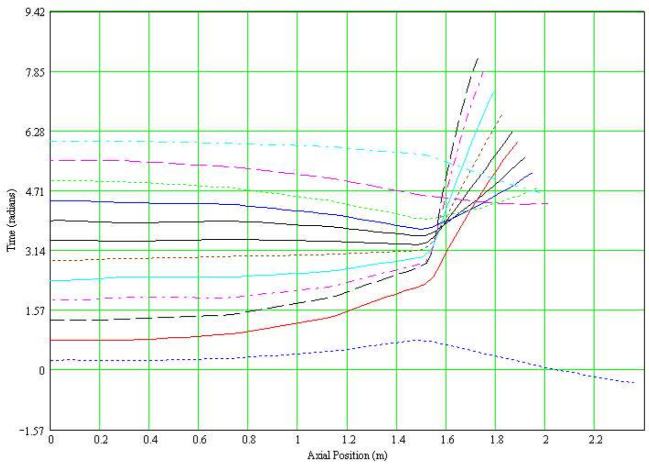
Gap number	1	2	3	4
Gap voltage (KV)	5	10	15	20
Initial gap field phase (rad)	0 and 2.05	-0.4	4.5	-1.2
Gap length (m)	0.029	0.024	0.049	0.053
Gap position (m)	0.1	0.298	0.7264	1.133
Frequency (MHz)	F and 2f	2f	f	f

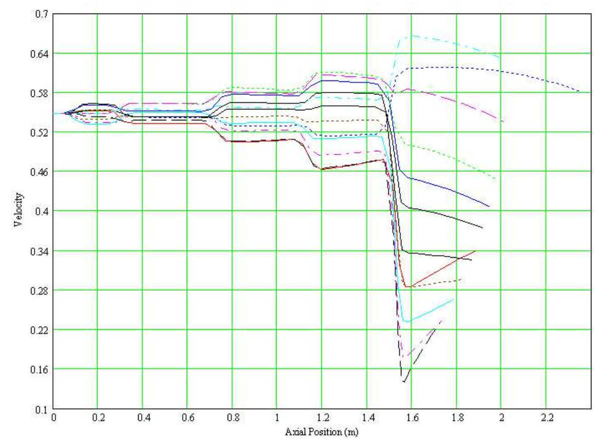


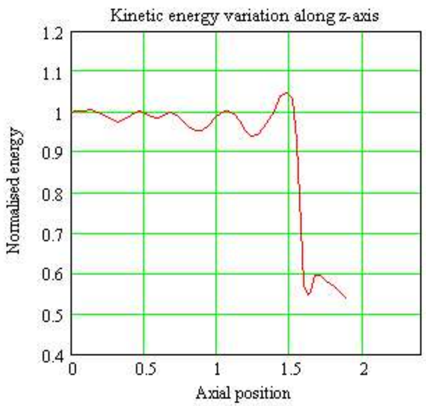


Bunching with output cavity

Gap number	1	2	3	4	5
Gap voltage (KV)	5	10	15	20	80
Initial gap field phase (rad)	0 and 2.05	-0.4	4.5	-1.2	-1.5
Gap length (m)	0.029	0.024	0.049	0.053	0.0375
Gap position (m)	0.1	0.298	0.7264	1.133	1.517
Frequency (MHz)	F and 2f	2f	f	f	f







Conclusion

•A spreadsheet for the investigation of beam bunching in a klystron has been developed using Mathcad.

•The results of gap voltage and gap phase of a klystron from a 2D ring model have been put into the spreadsheet for investigation. Applegate diagrams from the ring model and the Spreadsheet are similar, the efficiencies agree well for low input power.

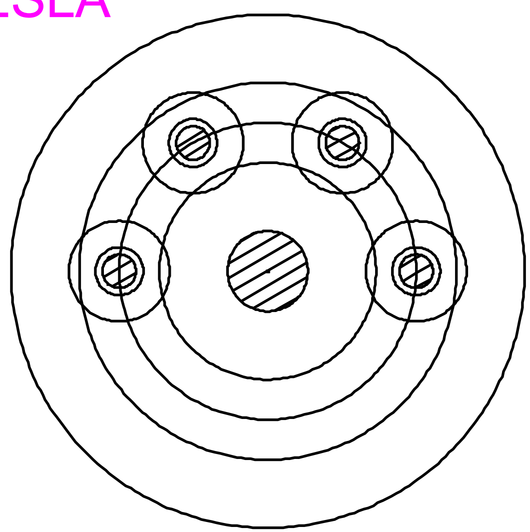
•Using the spreadsheet, a novel bunching technique is modelled, whereby a harmonic frequency is injected together with the fundamental frequency into the input cavity.

•The simulation results suggest a good result using the bunching method, including improved bunching, efficiency and decreased tube length.

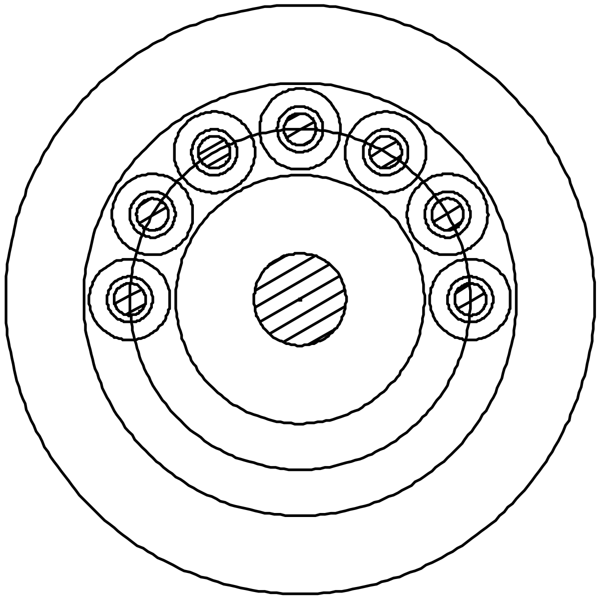
Preliminary design consideration for TESLA and CLIC

	Units	TESLA	CLIC
Linac RF frequency	GHz	1.3	0.937
RF pulse width	μsec	1500	100
Pulse repetition frequency	Hz	10	100
Klystron peak RF power	MW	10	50
Klystron mean RF power	KW	150	500
Duty cycle	%	1.5	1

TESLA



6 beams

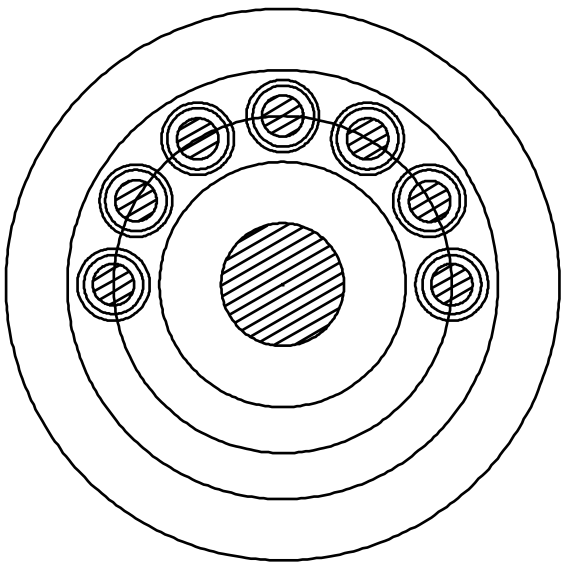


12 beams

TESLA

Parameter	Unit	6 beams	12 beams
Beam power (/beam)	MW	2.44	0.834
Beam Voltage	KV	111	89.3
Beam current	A	22	13.34
Electron wavelength	m	0.13	0.12
Beam radius	mm	4.18	3.47
Cathode loading	A/cm^2	5	5
Beam current density	A/cm^2	40	35.27
Area (radius) convergence		8 (2.8)	6.4 (2.53)
Cathode radius	mm	11.84	8.79
Drift tube radius	mm	5.97	4.95
Beam velocity	M/S	1.71×10^8	1.574
Relativity correct factor		1.217	1.175
Brillouin field	Tesla	0.043	0.039
Main focusing field	Tesla	0.086	0.079
Plasma wavelength	m	0.196	0.197
Plasma frequency reduction factor		0.108	0.108
Plasma wavelength with R	m	1.812	1.826

CLIC



Cavity	Cavity length	mm	90
	Cavity radius	mm	90
	Gap length	mm	18
	Inner conductor radius	mm	20
	Ridge inner radius	mm	40
	Ridge outer radius	mm	70
	Pitch circle diameter	mm	55
	Cavity frequency	GHz	0.959
	Characteristic impedance	Ω	20.0

50MW output power power

CLIC

Parameter	Unit	50MW	100MW
Beam power (/beam)	MW	5.950	11.9
Beam Voltage	KV	170	224
Beam current	A	35	53.1
Electron wavelength	m	0.212	0.23
Beam radius	mm	6.733	7.324
Cathode loading	A/cm^2	8	8
Beam current density	A/cm^2	24.6	31.5
Cathode radius	mm	11.80	14.534
Area (radius) convergence		3.07 (1.75)	3.94 (1.98)
Drift tube radius	mm	9.618	10.463
Beam velocity	M/S	1.98×10^8	2.156
Relativity correct factor		1.333	1.439
Brillouin field	Tesla	0.028	0.027
Main focusing field	Tesla	0.055	0.055
Plasma wavelength	m	0.347	0.38
Plasma frequency reduction factor		0.108	0.108
Plasma wavelength with R	m	3.217	3.52

CLIC

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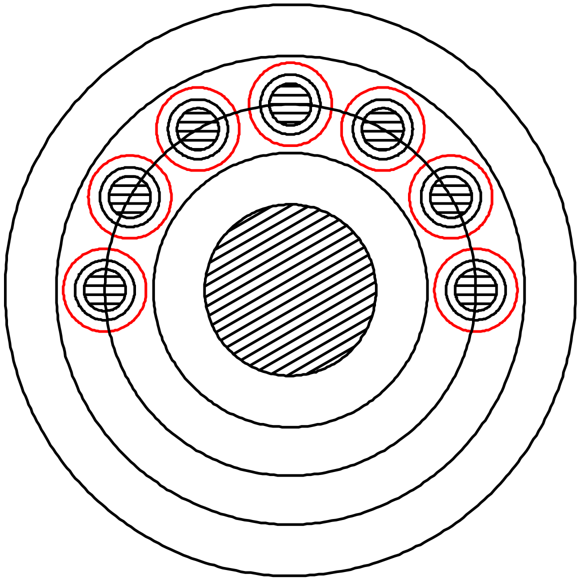
Cavity	Cavity length	mm	100
	Cavity radius	mm	100
	Gap length	mm	18.5
	Inner conductor radius	mm	30
	Ridge inner radius	mm	48
	Ridge outer radius	mm	82
	Pitch circle diameter	mm	65
	Cavity frequency	GHz	0.91996
	Characteristic impedance	Ω	16.14



100MW output power

The pitch diameter is enlarged for the arrangement of cathodes.