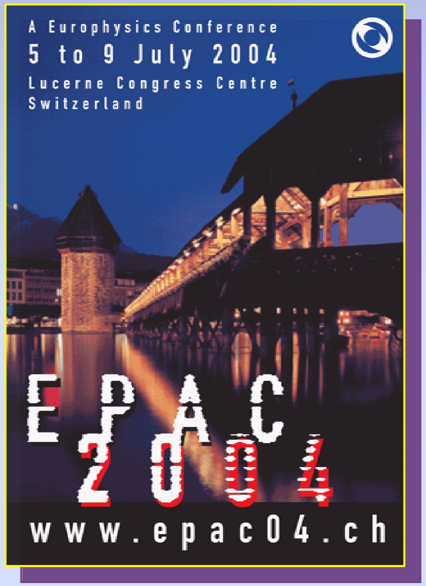
Emittance Control for Very Short Bunches



Karl Bane

Stanford Linear Accelerator Center

July 7, 2004 (modified for LBL talk Aug 13, 2004)

Thanks to P. Emma

Introduction

many recent accelerator projects call for the production of high energy bunched beams that are short, intense, and have small emittances

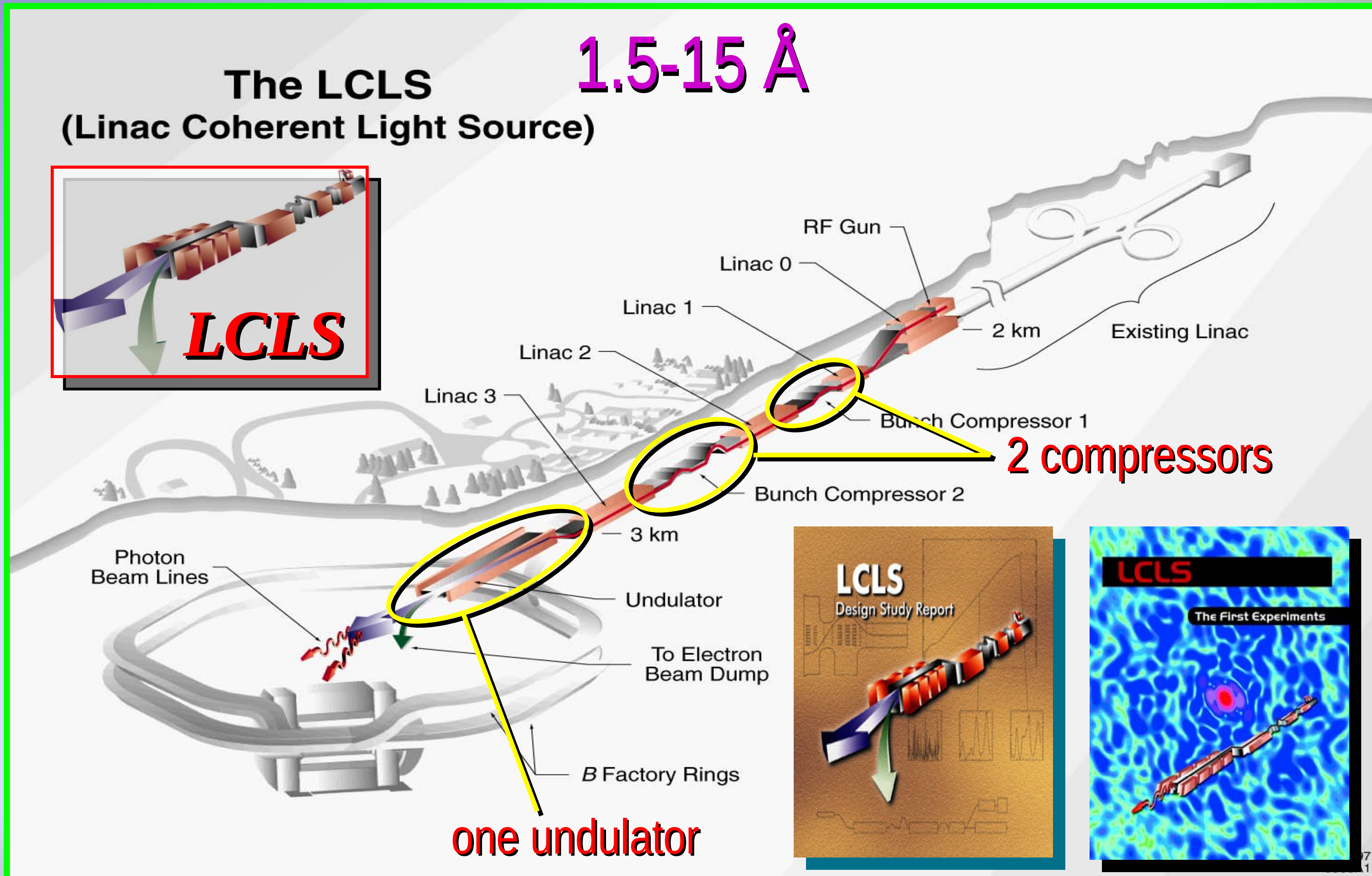
how do we quantify "short"? one simple answer is $\sigma_z/a \ll 1$ (σ_z bunch length, *a* beam pipe radius); in NLC main linac $\sigma_z/a = 0.02$, in LCLS SLAC linac $\sigma_z/a = 0.002$

"emittance control" can mean avoid unwanted emittance growth; can also mean "adjust" or "increase" in some situations

- will describe 4 wakes that are important for short bunches; focus on longitudinal plane, analytical expressions

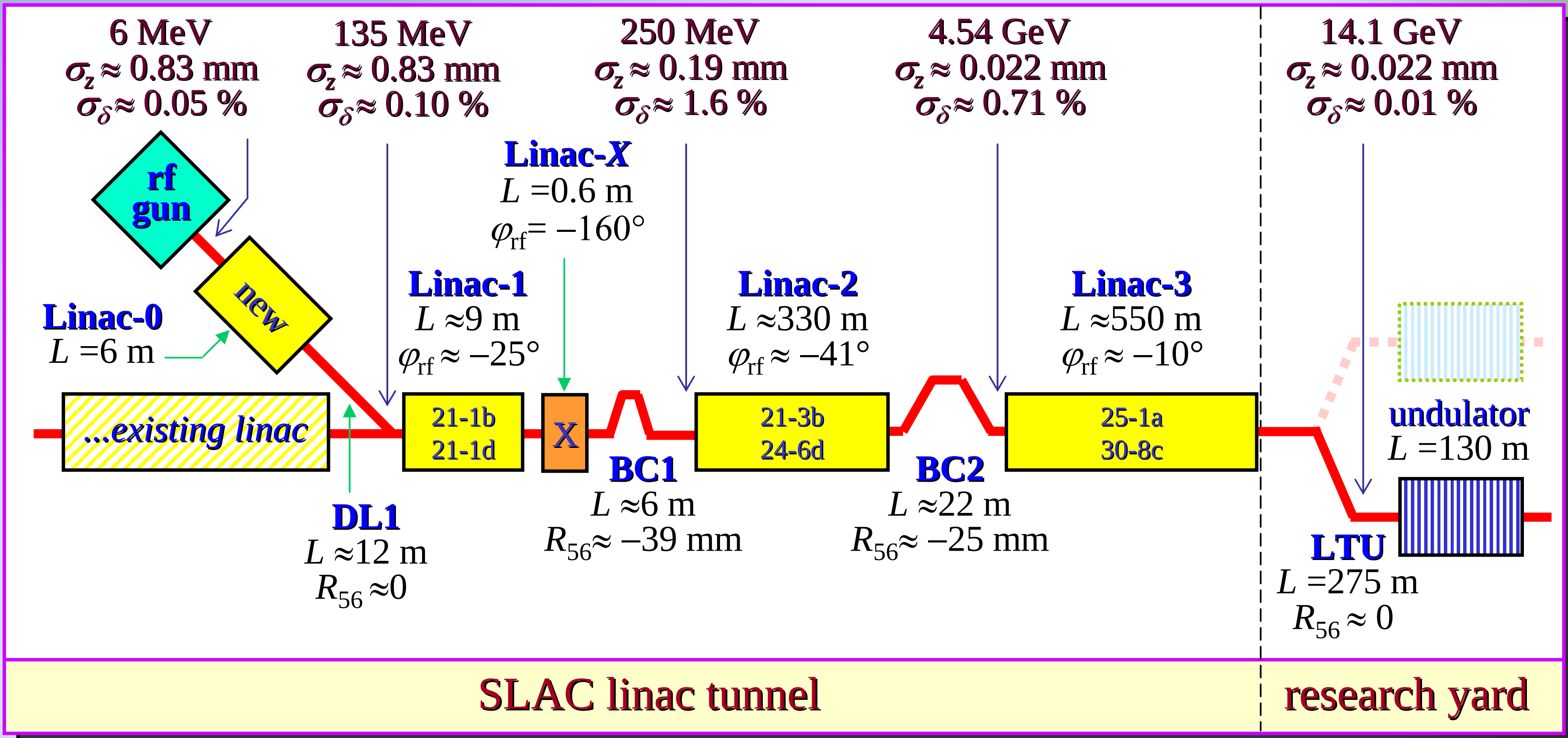
- will be applied to short-bunch regions of the LCLS, *spec.* for coherent synchrotron radiation (CSR) wake in the BC-2 chicane, accelerator structure wake in Linac-3, and resistive wall and roughness wakes in the undulator

LCLS at *SLAC*



X-FEL based on last 1-km of existing *SLAC* linac

LCLS Accelerator and Compressor Schematic



(Apr. 15, 2003)

Wakes and Impedances

- consider a particle, moving at speed *c* through a structure, that is followed by a test particle at distance *s*; Wake $W(s)$ is voltage loss (per structure or per period) experienced by the test particle; $W(s) = 0$ for $s<0$.

- bunch wake is voltage gain for a test particle in a distribution

$$\mathcal{W}(s) = -\int_0^\infty W(s')\lambda_z(s-s')\,ds' \ .$$

average of minus bunch wake $-\langle\mathcal{W}\rangle$ is loss factor; energy spread increase $\delta E_{rms} = eNL\mathcal{W}_{rms}$, with eN charge, L length of structure (in periodic case).

- impedance

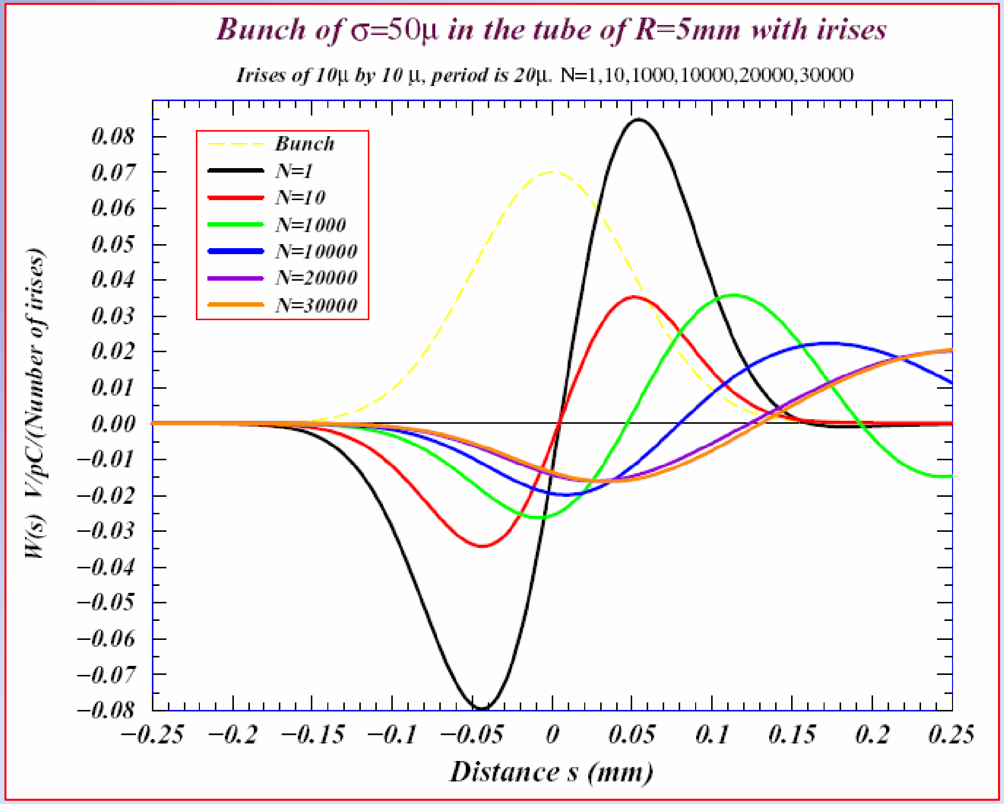
$$Z(k) = \int_0^\infty W(s)e^{iks}ds \ ,$$

- similar for transverse: W_x, Z_x

Considerations for Short Bunches

catch-up distance: wake is typically taken to act instantaneously. If head particle passes e.g. the beginning of a cavity, tail particle doesn't know it until $z = a^2/2s$ (a beam pipe radius, s separation of particles) later. If a= 1cm and s= 20 μm, then z= 2.5 m.

transient region: similarly, for periodic structures, there will be a transient regime before steady-state is reached; for Gaussian with length σ_z, transient will last until $z \approx a^2/2\sigma_z$



Simulation of wake per period generated by a bunch in a tube with *N* small corrugations (A. Novokhatski).

limiting value of wake: for periodic, cylindrically symmetric structures whose closest approach to axis is *a*, the steady-state wakes have the property

$$W(0^+) = \frac{Z_0 c}{\pi a^2} \quad \text{and} \quad W_x'(0^+) = \frac{2 Z_0 c}{\pi a^4} \ ,$$

with $W_x(0^+) = 0$, where $Z_0 = 377\ \Omega$.

— this is true for a resistive pipe, a disk-loaded accelerator structure, a pipe with small periodic corrugations, and a dielectric tube within a pipe; it appears to be a general property

— for very short bunches the longitudinal wake approaches a maximum, the transverse wake zero

finite energy: impedance drops sharply to 0 when $k > \gamma/a$ (γ Lorentz energy factor); for $\sigma_z < a/\gamma$, replace σ_z by a/γ in wake formulas; if $a = 1$ cm, energy $E = 14$ GeV, this occurs when $\sigma_z = 0.4$ μm.

A. Resistive Wall Wake

•Dc conductivity

•impedance (see A. Chao): $$Z = \left(\frac{Z_0}{2\pi a}\right)\frac{1}{\frac{\lambda}{k} - \frac{ika}{2}}$$

with $$\lambda = \sqrt{\frac{2\pi\sigma|k|}{c}}\,[i + sgn(k)]$$

•inverse Fourier transform to find wake

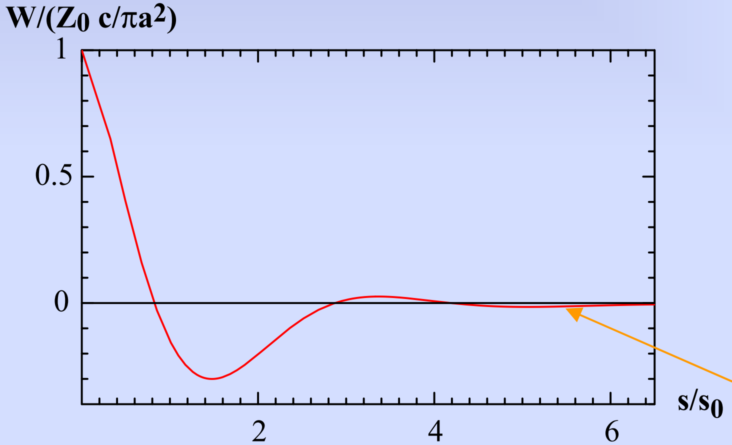
•general solution is composed of a resonator term and a diffusion term

General solution

$$W = \frac{4Z_0 c}{\pi a^2}\left(\frac{e^{-s/s_0}}{3}\cos\frac{\sqrt{3}s}{s_0} - \frac{\sqrt{2}}{\pi}\int_0^\infty \frac{dx\, x^2 e^{-x^2 s/s_0}}{x^6+8}\right)$$

$$s_0 = \left(\frac{2a^2}{Z_0\sigma}\right)^{\frac{1}{3}}$$

(for Cu with *a*= 2.5mm, s_0= 8.1μm)



long range wake:

$$W(s) = -\frac{c}{4\pi^{3/2}a}\sqrt{\frac{Z_0}{\sigma}}\frac{1}{s^{3/2}}\ ,$$

wake for dc conductivity

•*Ac conductivity*

•resistive wall wake is a limiting effect in the LCLS undulator, with the induced ΔE~ ρ the Pierce parameter (=0.05%)

•can add effects of ac conductivity (see K. Bane and M. Sands, SLAC-PUB-95-7074) to resistive wall model

Free electron model of conductivity (see e.g. Ashcroft and . Mermin, *Solid State Physics*)

•Drude free-electron model of conductivity (1900): conduction electrons are treated as an ideal gas, whose velocity distribution is given in equilibrium at temperature *T* by the Maxwell-Boltzmann distribution

•Sommerfeld (1920's) replaced the distribution by the Fermi-Dirac distribution

•this free-electron model correctly describes many electrical and thermal properties of metals

Parameters

- density of conduction electrons n (~10^{22}/cm^3)
- collision time (or mean free time, or relaxation time) τ (~10^{-14} s)
- dc conductivity $\sigma = ne^2\tau/m$
- ac conductivity $\tilde{\sigma} = \dfrac{\sigma}{1 - i\omega\tau}$

=>not consistent to ignore ac conductivity

- Fermi velocity v_F (~0.01c)
- mean free path $\ell = v_F\tau$
- note that σ/τ, ℓ/τ nearly independent of temperature

How good is the free electron model for real metals?

Im(ε) for Cu

Im(ε) for Ag

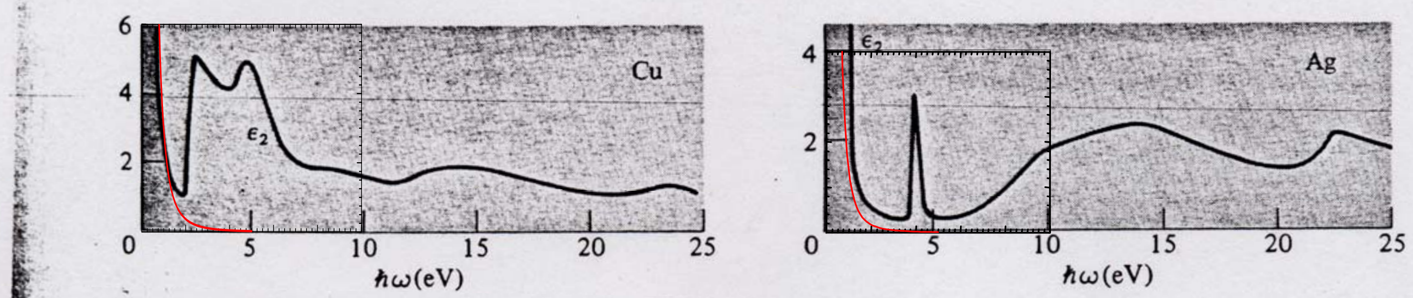


Figure 15.12
The imaginary part of the dielectric constant, $\epsilon_2(\omega) = \mathrm{Im}\,\epsilon(\omega)$ vs. $\hbar\omega$, as deduced from reflectivity

Im(ε) from reflectivity measurements (Ashcroft/Mermin, p. 297)

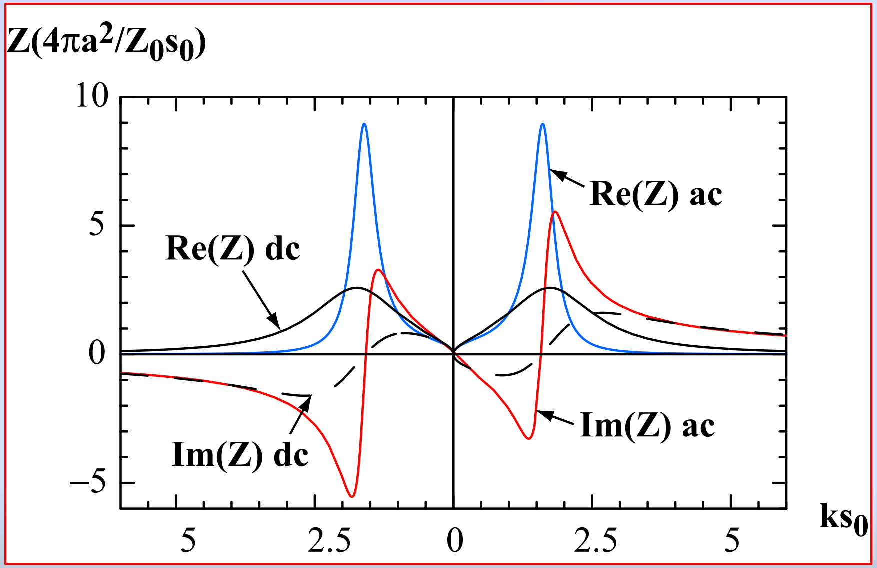
- note: $\epsilon(\omega) = 1 + \frac{4\pi i\tilde{\sigma}}{\omega}$ so $Im(\epsilon) = \frac{4\pi\sigma}{\omega}\frac{1}{(1+\omega^2\tau^2)}$
- k= 1/0.1μm⇔{ω=2eV, red light

Impedance

- new parameter $\Gamma = c\tau/s_0$.

- for Cu with beam pipe radius a= 2.5 mm, s_0= 8 μm, $c\tau$= 8 μm, Γ= 1.0; for Al, s_0= 9.3 μm, $c\tau$= 2.4 μm, Γ= 0.26.

- for ac conductivity replace σ with $\tilde{\sigma}$ in parameter λ; then again take inverse Fourier transform of Z for wake



note: Re(Z)~0 for k≥ 1/4μm

impedance

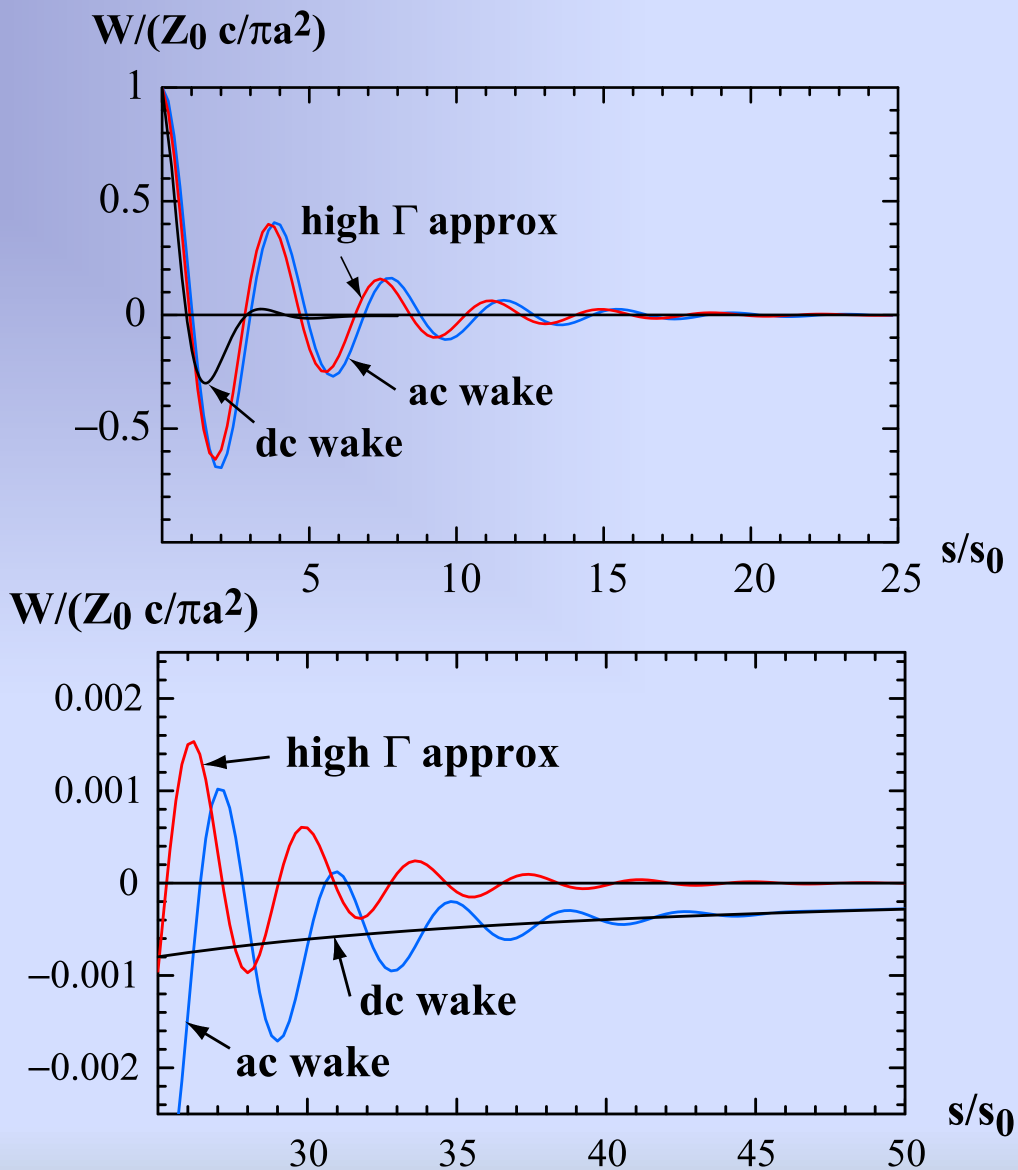
•wake again $W_z(z)$ is composed of a resonator and a diffusion component

•for $\Gamma \gtrsim 1$, can approximate

$$W_z(s) = \frac{Z_0 c}{\pi a^2} e^{-s/4c\tau} \cos\left[\sqrt{\frac{2\omega_p}{ac}} s\right]$$

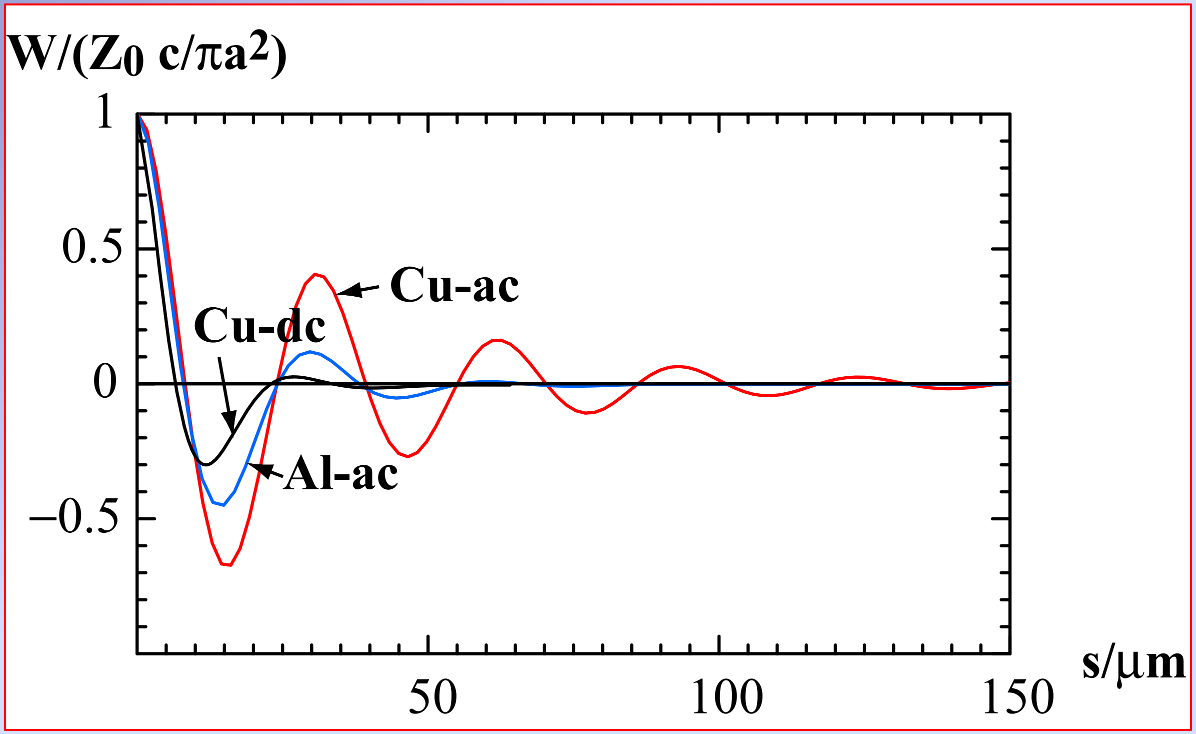
with the plasma frequency $\omega_p = \sqrt{4\pi\sigma/\tau}$

for LCLS with Cu, plasma wave number k_p= 1/0.02μm; mode wave number k_r= 1/5μm, damping time $c\tau_r$= 32μm

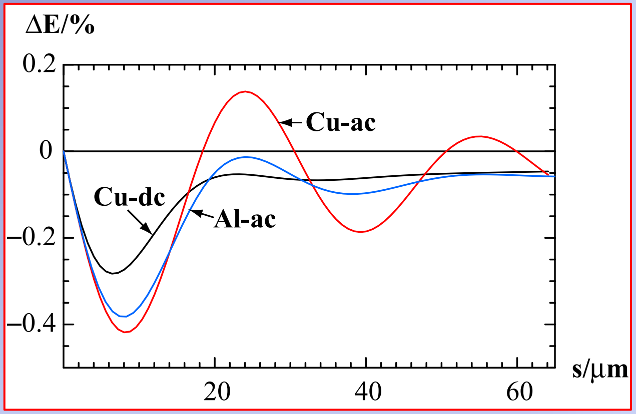


s_0= 8 μm

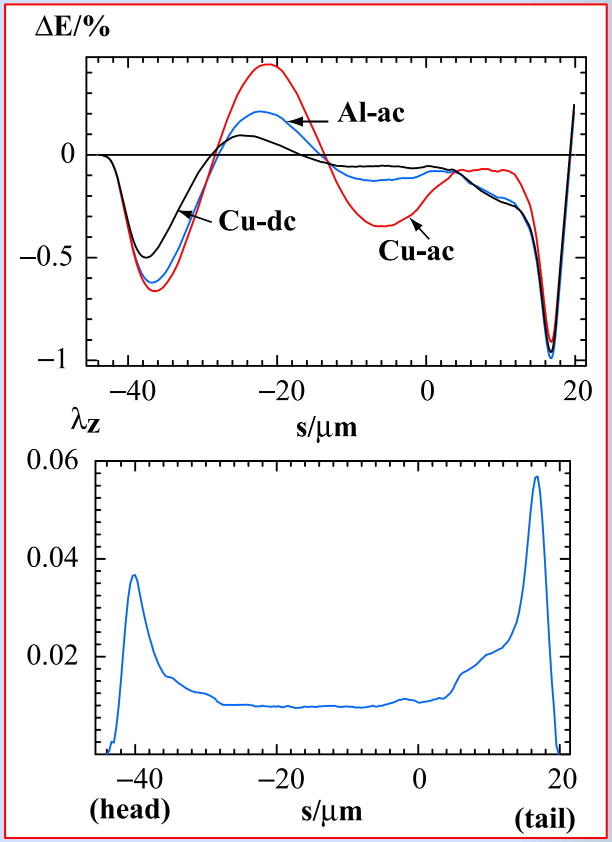
ac wake with high Γ approximation



Point charge wake



Induced energy change for rectangular bunch with full length of 65 μm (note Pierce parameter ρ= 0.05%)



charge—1 nC, energy—14 GeV, tube radius—2.5 mm, tube length—130 m

Induced energy change (top) for LCLS bunch shape (bottom).

•***Anomalous skin effect*** (Reuter and Sondheimer)

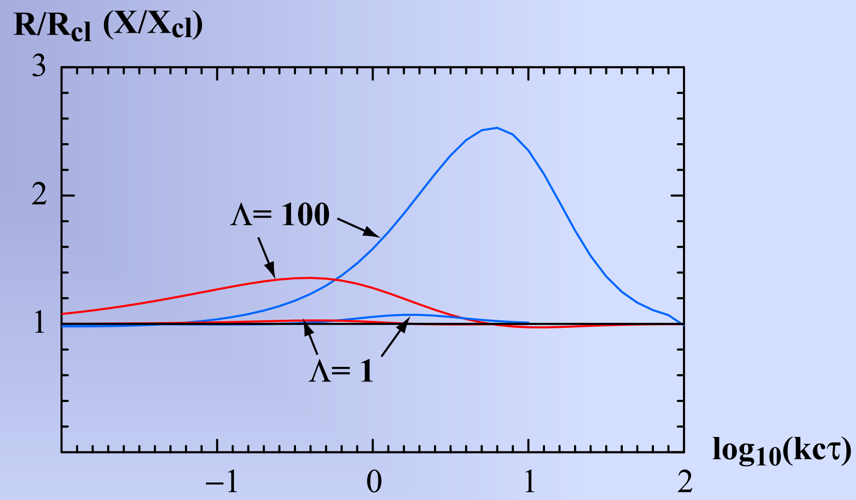
when $\ell > \delta = c/\sqrt{2\pi\omega\sigma}$ the skin depth, the anomalous skin effect occurs, the fields don't drop exponentially with distance into metal

in principle this can happen at low temperatures or high frequencies; nevertheless, "It is evident that no appreciable departure from the classical behaviour is to be expected at ordinary temperatures, so that the anomalous skin effect is essentially a low-temperature phenomenon"—Reuter and Sondheimer.

for Cu at room temperature, ℓ= 0.04μm and for k= 1/20μm, δ= 0.04μm

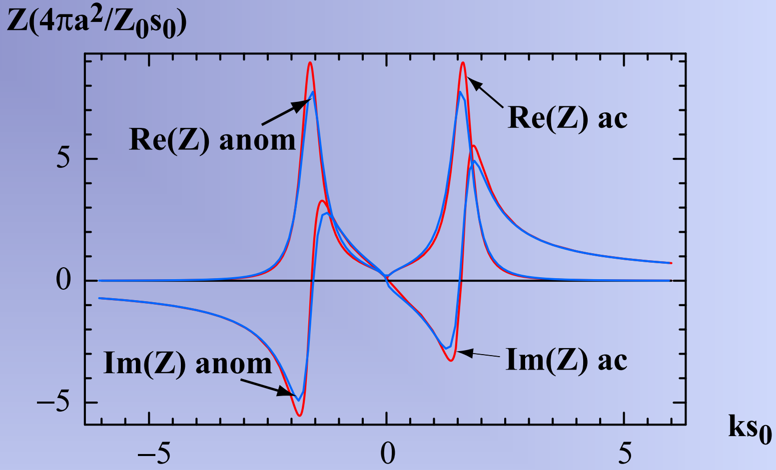
ASE parameter $\alpha=1.5\ell^2/\delta^2$; normalized parameter $\Lambda = \alpha/kc\tau$; for Cu at room temperature Λ= 3.4

results given in terms of surface impedance $Z_s = R+iX$, compared to classical (ac) surface impedance $(Z_s)_{cl} = R_{cl}+iX_{cl}$



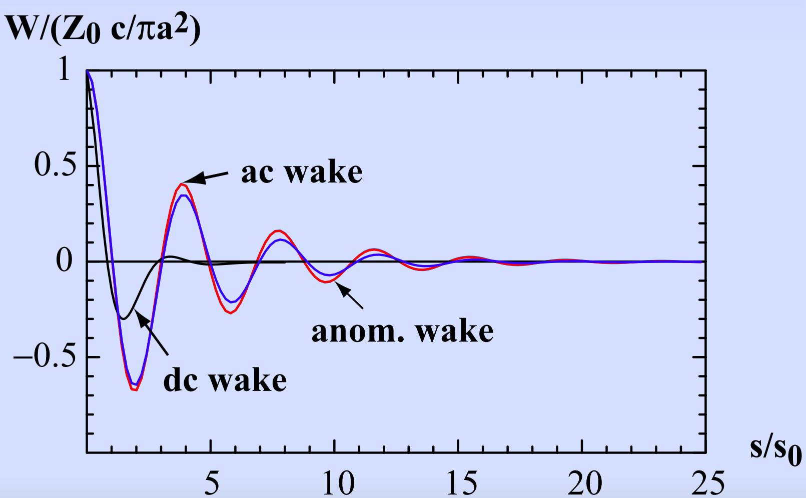
sensitivity of anomalous skin effect to frequency; given are R/R_{cl} (blue) and X/X_{cl} (red).

- note: for LCLS $\Lambda = 3.4$, peak of $R/R_{cl} = 1.2$
- to find impedance, set $\lambda = \dfrac{4\pi|k|/c}{R\,\mathrm{sgn}(k) - iX}$ in Z; for wake again take inverse Fourier transform



impedance for a=2.5mm Cu tube including anomalous skin effect

Λ= 3.4



wake

- Affect/implications of r-w wake on LCLS still under study

B. Accelerator Structure Wake

- for short bunch ($\sigma/a \ll 1$) passing through a single cavity

$$W(s) = \frac{Z_0 c}{\sqrt{2}\pi^2 a}\sqrt{\frac{g}{s}} \ ,$$

where g is gap; impedance varies $Z \sim k^{-1/2}$

- for periodic structure with period p, high frequency impedance

$$Z(k) \approx \frac{iZ_0}{\pi k a^2}\left[1 + (1+i)\frac{\alpha(g/p)\,p}{a}\left(\frac{\pi}{kg}\right)^{1/2}\right]^{-1} \ ,$$

with

$$\alpha(x) \approx 1 - 0.465\sqrt{x} - 0.070x$$

$\mathrm{Re}(Z) \sim k^{-3/2}$

[Gluckstern; Yokoya and Bane]

- numerical calculation of wake can be fit to (over useful parameter range)

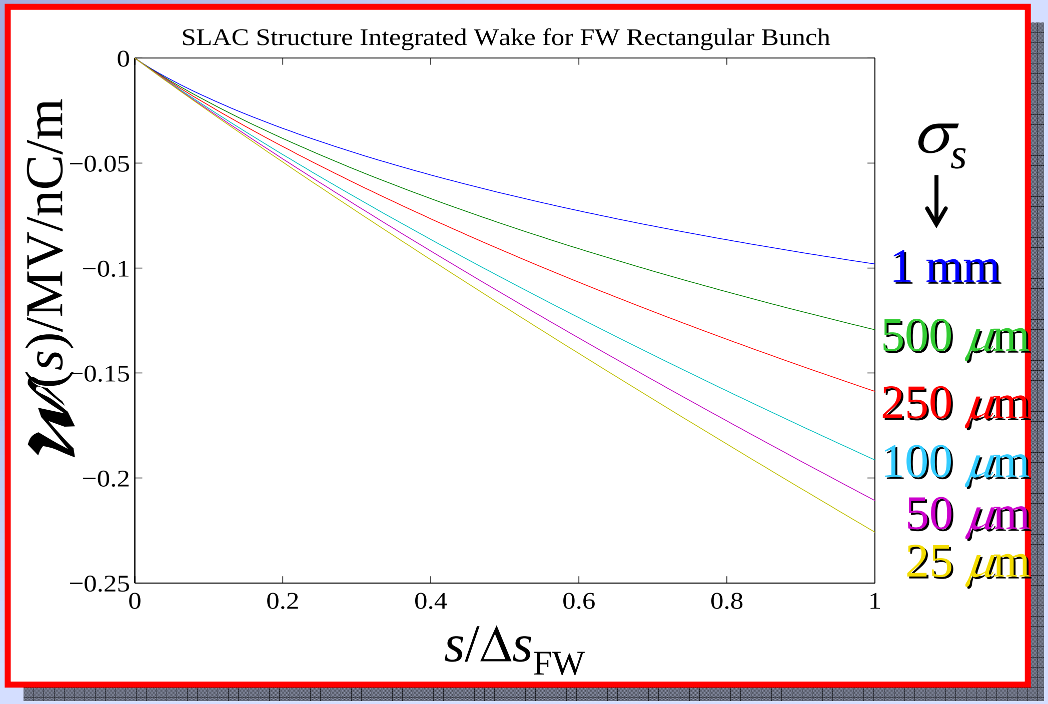
$$W(s) = \frac{Z_0 c}{\pi a^2} \exp\left(-\sqrt{s/s_1}\right) \quad \text{with} \quad s_1 = 0.41 \frac{a^{1.8} g^{1.6}}{p^{2.4}} .$$

in SLAC linac, s_1=1.5 mm

- for LCLS Linac-3, σ_z= 20 μm, *W*~ constant; note transient regime $z \sim a^2/2\sigma_z \sim$ 3.4 m (small compared to 550 m)

- same has been done for transverse wake

[K. Bane, et al]

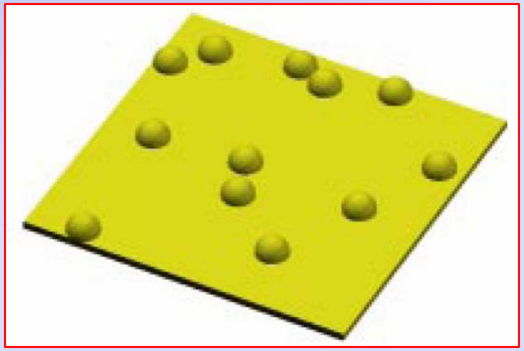


Bunch wake for a rectangular bunch distribution

C. Roughness Impedance

A metallic beam pipe with a rough surface has an impedance that is enhanced at high frequencies. Two approaches to modeling are (i) random collection of bumps, (ii) small periodic corrugations

(i) Random bumps



Impedance of one hemispherical bump (of radius *h*) for $k \ll 1/h$

$$Z(k) = ikc\mathcal{L}_1 = ik\frac{Z_0 h^3}{4\pi a^2} ,$$

[S. Kurennoy]

- for many bumps (α filling factor, *f* form factor)

$$\mathcal{L}/L = \frac{2\alpha f a \mathcal{L}_1}{h^2} = \frac{\alpha f Z_0 h}{2\pi a c} \ ,$$

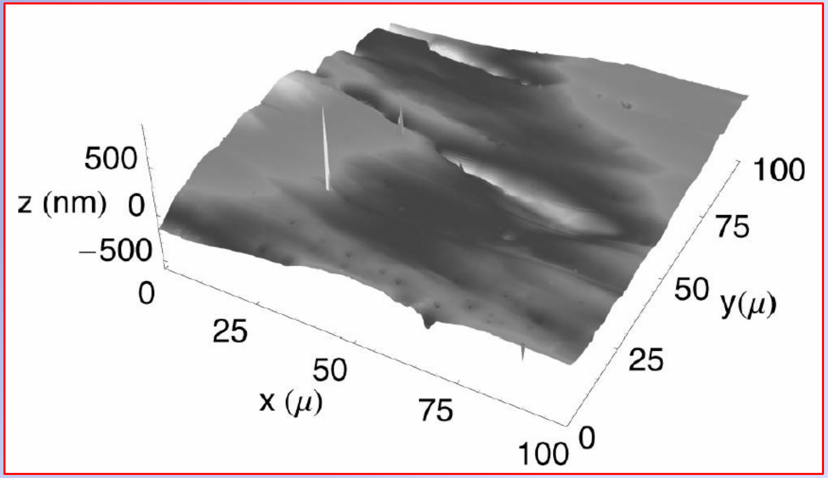
- idea has been systematized so that, from surface measurement, can find impedance:

$$\mathcal{L}/L = \frac{Z_0}{2\pi c a} \int_{-\infty}^{\infty} \frac{k_z^2}{\sqrt{k_\theta^2 + k_z^2}} S(k_z, k_\theta)\, dk_z dk_\theta \ ,$$

with S spectrum of surface, k_z, k_θ, longitudinal, azimuthal wave numbers

- bunch wake ~λ_z'; for Gaussian $\mathcal{W}_{rms} \approx 0.06 c^2 \mathcal{L}/L \sigma_z^2$; can't use model for rectangular or other non-smooth distribution

[K. Bane, et al; G. Stupakov]



Sample profile measured with atomic force microscope
[from G. Stupakov, et al]

- Note: variation along surface is more gradual than variation perpendicular to surface

(ii) Small periodic corrugations

motivation: numerical simulations of many randomly placed, small cavities on a pipe found that, in steady state, the short range wake is very similar to truly periodic case

- consider a beam pipe with small corrugations of height *h*, period *p*, and gap *p*/2. If $h/p \gtrsim 1$, wake

$$W(s) \approx \frac{Z_0 c}{\pi a^2} \cos k_0 s \quad \text{with} \quad k_0 = \frac{2}{\sqrt{ah}} .$$

- for Gaussian, with $k_0\sigma_z \gg 1$, becomes inductive with $\mathcal{L}/L = Z_0 h/(4ac)$, similar to earlier model

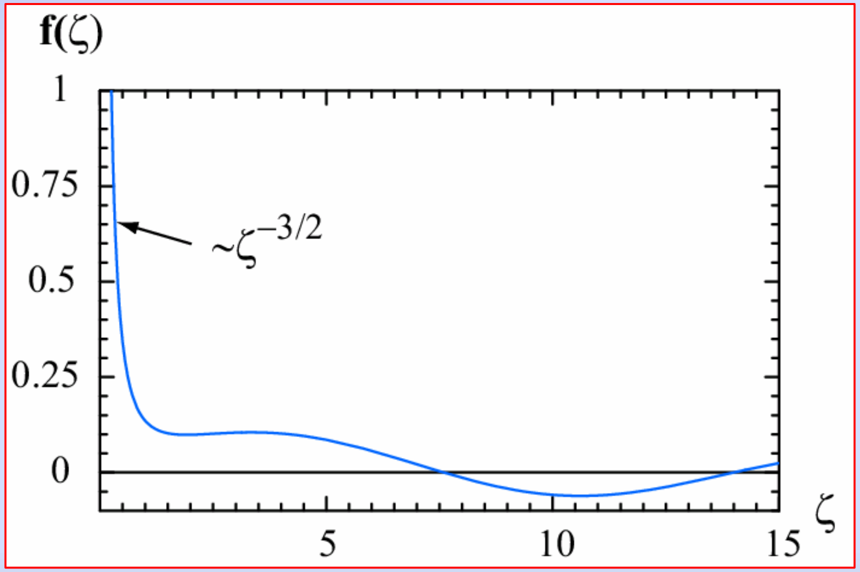
- can be used with non-smooth bunch distribution

[A. Novokhatski, et al; K. Bane and A. Novokhatski]

- as h/p becomes small, low frequency mode becomes many weak, closely spaced modes $k \approx \pi/p$; for $h/p \ll 1$ wake

$$W(s) = \frac{Z_0 c h^2 k_1^3}{4\pi a} f(k_1 s) \ , \quad f(\zeta) = -\frac{1}{2\sqrt{\pi}} \frac{\partial}{\partial \zeta} \frac{\cos(\zeta/2) + \sin(\zeta/2)}{\sqrt{\zeta}}$$

with $k_1 = 2\pi/p$



[G. Stupakov]

- for $k_1 s \lesssim 1$ (but not too small):
  - $W \sim s^{-3/2}$; for bunch $\mathcal{W} \sim \sigma_z^{-3/2}$
  - bunch wake weaker by ~h/p than single mode model
  - wake behaves like metal, with effective $\sigma = 16/(Z_0 h^4 k_1^3)$

- for LCLS, if we assume (earlier displayed) measured surface profile is representative of undulator beam pipe ($h \sim 0.5$ µm, $p \sim 100$ µm) and $\sigma_z = 20$ µm, then this model applies, and

$\Rightarrow$roughness wake 0.15 as strong as resistive wall wake (with Cu)

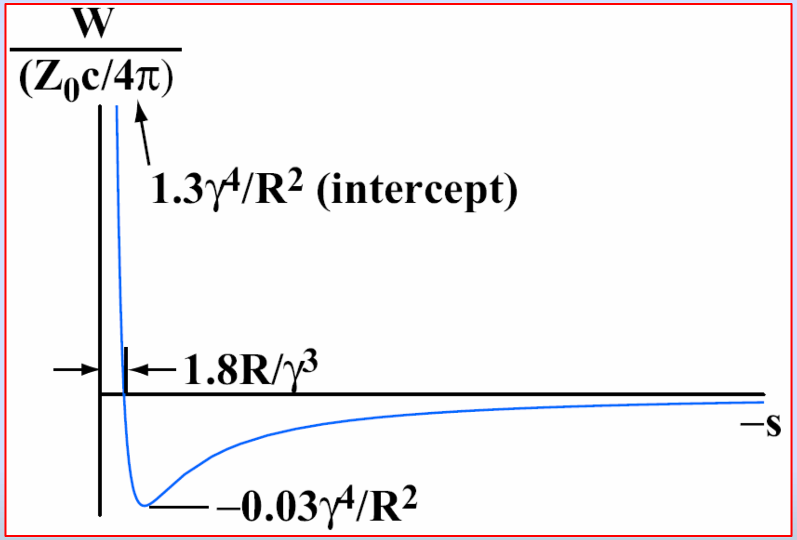
- some measurements have been done (DESY, Brookhaven) but more needed

D. CSR Wake

- CSR effect on bunch can be described in terms of wakefield. Consider ultra-relativistic particle (and test particle) moving on circle of radius R in free space. For $(-s) \gg R/\gamma^3$

$$W(s) = -\frac{Z_0 c}{2 \cdot 3^{4/3} \pi R^{2/3} (-s)^{4/3}} \qquad s < 0 \ ,$$

while $W(0^-) = Z_0 c \gamma^4 / (3\pi R^2)$



[J. Murphy, et al; Y. Derbenev, et al]

- unlike normal wake, only nonzero when test particle is ahead of exciting charge ($s < 0$)

- for a bunch wake scales $\sim R^{-2/3}\sigma_z^{-4/3}$

- impedance

$$Z(k) = \frac{Z_0}{2 \cdot 3^{1/3}\pi}\Gamma\left(\frac{2}{3}\right) e^{i\pi/6}\frac{k^{1/3}}{R^{2/3}} \ ,$$

with $\Gamma(2/3)= 1.35$; valid to high frequencies ($k\sim \gamma^3/R$)

- shielded by beam pipe if $\sigma_z/a \gtrsim (a/R)^{1/2}$; for BC2 of LCLS $\sigma_z= 20$ µm, $a= 1$cm, $R= 15$ m $\Rightarrow$ bunch is 13 times too short for shielding

- on entering a bend, the distance of transient wakes is $z\approx (24R^2\sigma_z)^{1/3}$; for above example transient $z= 0.5$ m

[J. Murphy, et al; Y. Derbenev, et al; R. Warnock]

- Chicane compressors are composed of 3 or 4 bends separated by drifts. One can consider the potential energy change (the "compression work") that beam undergoes in being compressed. If compression factor is large (assuming Gaussian bunch) this is equivalent to an average kinetic energy change

$$\langle \delta E \rangle = -\frac{e^2 N Z_0 c}{4\pi^{3/2}\sigma_z} \ln\left(\frac{\gamma\sigma_z}{\sigma_x + \sigma_y}\right)$$

where beam sizes are final quantities, and the rms spread $\delta E_{rms} \approx -0.4 \langle \delta E \rangle$

[M. Dohlus; K. Bane and A. Chao]

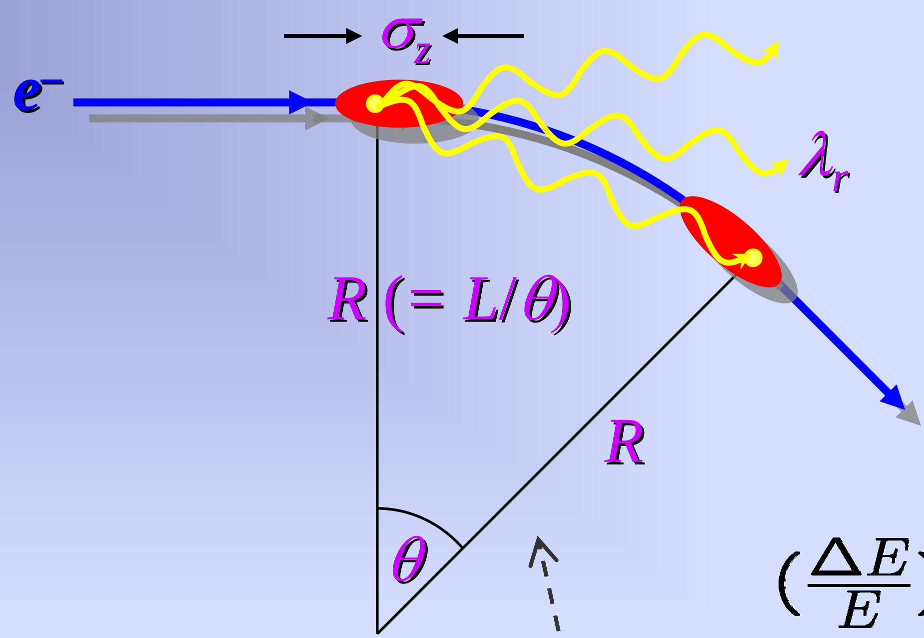
- to simulate CSR force in a chicane, computer programs slice the beam into macro-particles, and solve the Lienard-Wiechert potentials

- bunch can have transverse dimensions, shielding can be added, can be self-consistent; the programs typically are time consuming to run.

- analytical solutions of 1D wake of particle entering, traversing, and leaving a bend without shielding have been derived (includes transients); when used in a 1D tracking program, they are quick to calculate and seem to agree reasonably well for typical parameters

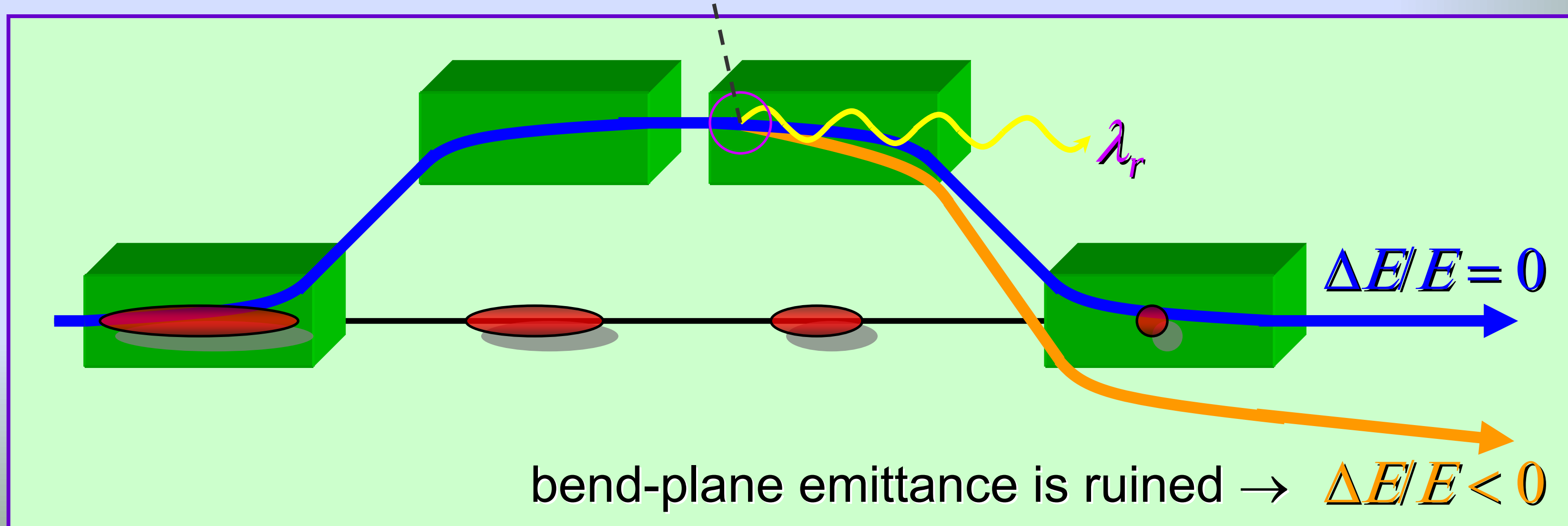
[E. Saldin, et al]

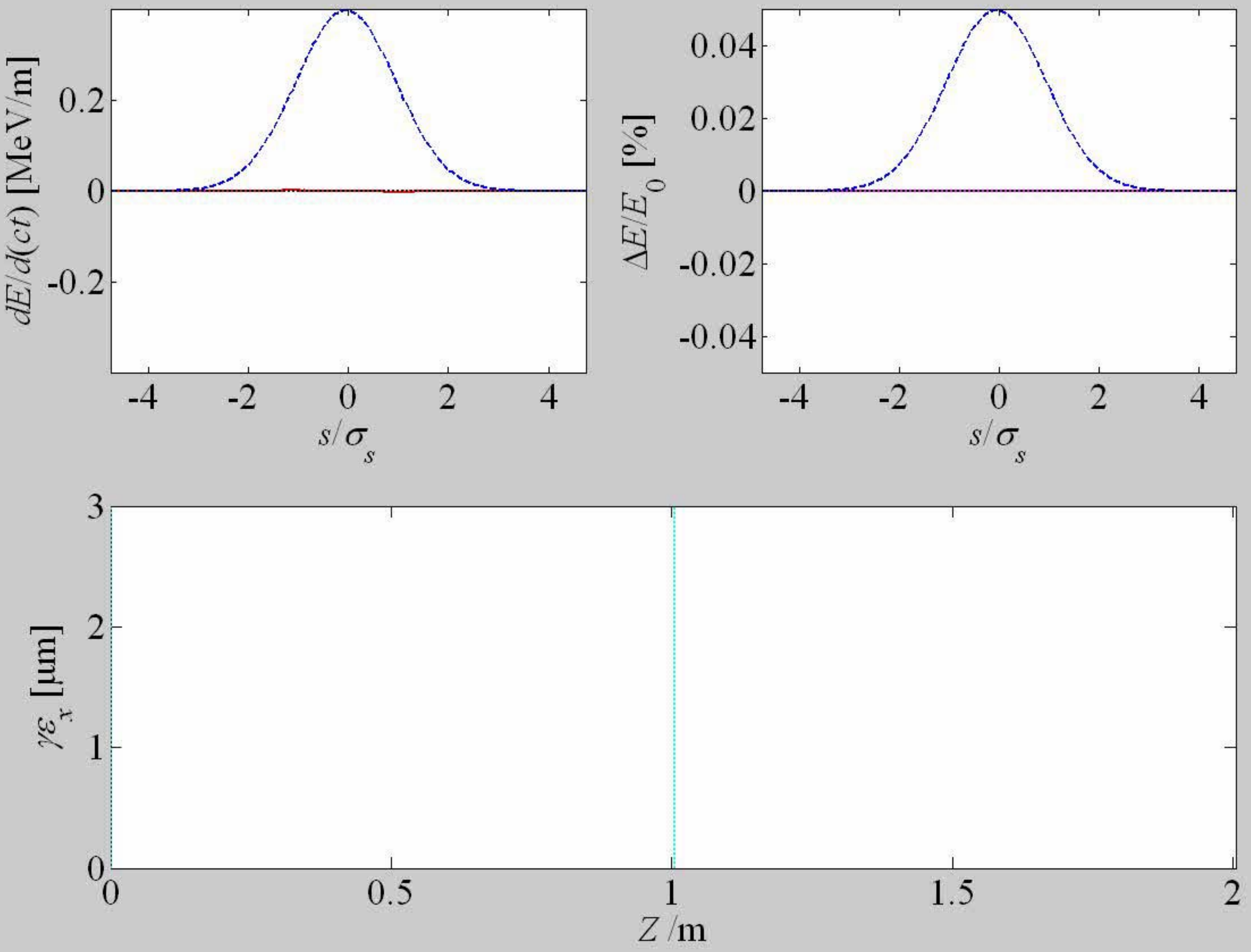
Coherent Synchrotron Radiation in Bends



Coherent radiation for $\lambda_r > \sigma_z$

$$\left(\frac{\Delta E}{E}\right)_{rms} \approx 0.22 \frac{r_e N L}{\gamma R^{2/3} \sigma_z^{4/3}}$$





$dE/d(ct)$ [MeV/m]
0.2
0
-0.2
-4
-2
0
2
4
s/σ_s
$\Delta E/E_0$ [%]
0.04
0.02
0
-0.02
-0.04
-4
-2
0
2
4
s/σ_s
$\gamma\varepsilon_x$ [μm]
3
2
1
0
0
0.5
1
1.5
2
Z /m

TABLE V. List of benchmarked codes and of the beam parameters at the end of the chicane. We have indicated with δE the relative energy loss and with $\delta\sigma_E$ the change in the relative energy spread.

Dimension	Code Name	δE (%)	$\delta\sigma_E$ (%)	ε
3D	TRAFIC4	−0.058	−0.002	1.4
3D	TREDI	−0.041	0.017	2.3
2D	Program by Li	−0.056	−0.006	1.32
1D line charge	ELEGANT	−0.045	−0.0043	1.55
1D line charge	CSR_CALC (Emma)	−0.043	−0.004	1.52
1D line charge	Program by Dohlus	−0.045	−0.011	1.62

Comparison of results from different CSR programs for the so-called Berlin benchmark chicane (from report of L. Giannessi).

- potential energy formula gets $\langle \delta E \rangle / E = -0.051\%$; $\delta E_{rms}/E = 0.020\%$

[A. Kabel, et al; L. Giannessi; R. Li; M. Borland; P. Emma; M. Dohlus]

Emittance Control

Slice emittance:

in the NLC the projected emittance is most important; in LCLS slice emittance (emittance over slippage length) is most important (in LCLS, 0.5 μm vs. σ_z= 20 μm); wakes only weakly affect slice emittance directly

a compressor, in principle, can couple head-tail effects into slice emittance [see poster MOPKF81, A. Kabel, P. Emma]

forces that can affect slice emittance directly are *e.g.* space charge, incoherent synchrotron radiation, intra-beam scattering

LCLS example

consider wake effects in LCLS BC-2, Linac-3, undulator: eN= 1 nC, bunch shape uniform with σ_z= 20 µm; before Linac-3, E= 4.5 GeV, after E= 14 GeV; length of Linac-3, L= 550 m, of undulator, L= 130 m.

Linac-3

— effect of transverse wake: $W_x \approx 2Z_0 cs/(\pi a^4)$; due to betatron oscillation $\delta\varepsilon/\varepsilon \approx \upsilon^2/2$ with $\upsilon = e^2 NL \langle \mathcal{W}_x \rangle \beta/(2E)$= 0.06 $\Rightarrow$ $\delta\varepsilon/\varepsilon$ is insignificant

— longitudinal wake is used to take out residual chirp after BC-2: $W \approx Z_0 c/(\pi a^2)$; induced chirp is almost linear with $\delta E_{rms}/E = e^2 N \mathcal{W}_{rms} L/E = 0.3\%$.

undulator

resistive wall wake dominates over roughness wake, and $\delta E_{rms}/E = e^2 N \mathcal{W}_{rms} L/E = 0.05\%$; needs to be less than Pierce parameter $\rho = 5\times10^{-4}$;
$\Rightarrow$ near limit of acceptability

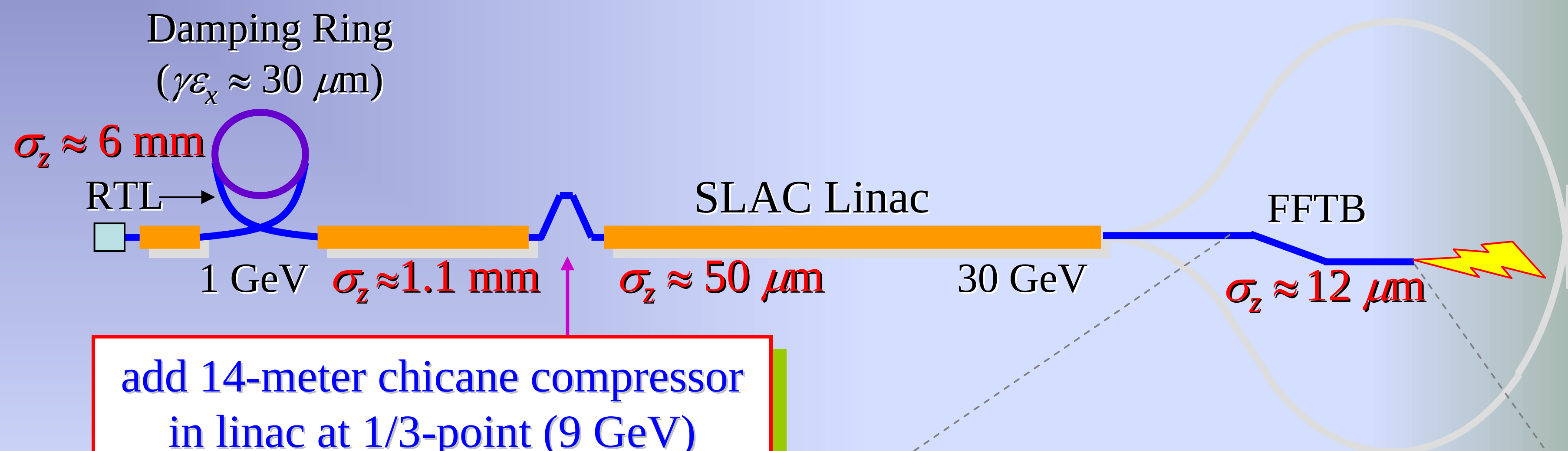
compressor BC-2

1D simulation (with Gaussian) yields $\delta E_{rms}/E = 0.018\%$, leading to $\delta\varepsilon/\varepsilon = 38\%$; potential energy equation yields $\delta E_{rms}/E = 0.016\%$

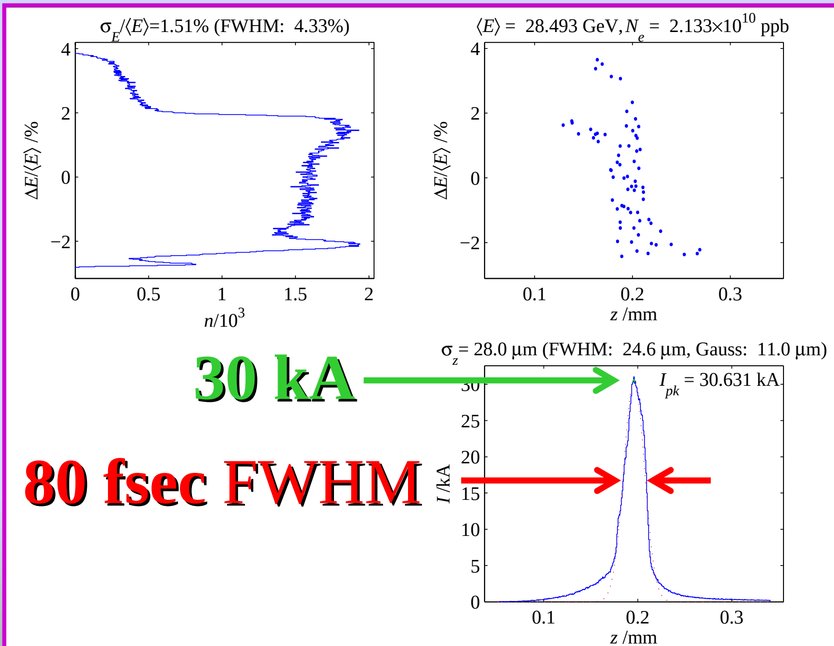
- microbunch instability driven by CSR or longitudinal space charge impedance is an important “short bunch” effect in the LCLS

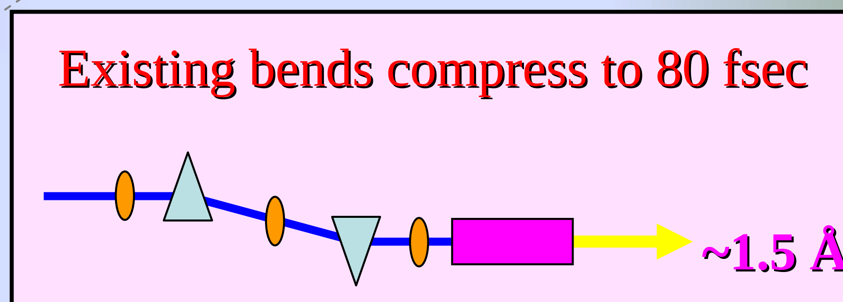
- emittance control can also mean increasing emittance, *e.g.* using a laser to heat the beam to suppress a longitudinal space charge induced microbunch instability; using a thin beryllium, slotted foil in the middle of BC-2 to spoil emittance of most particles, in order to shorten the light pulse

Short Bunch Generation in the SLAC Linac



add 14-meter chicane compressor in linac at 1/3-point (9 GeV)



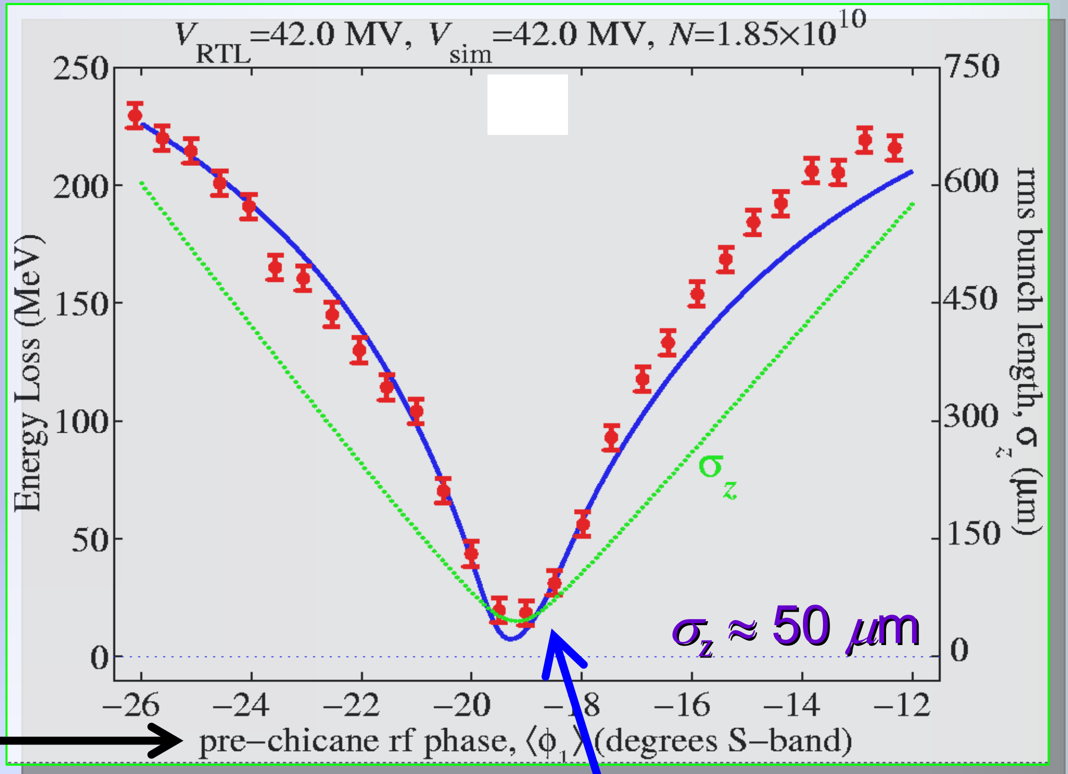


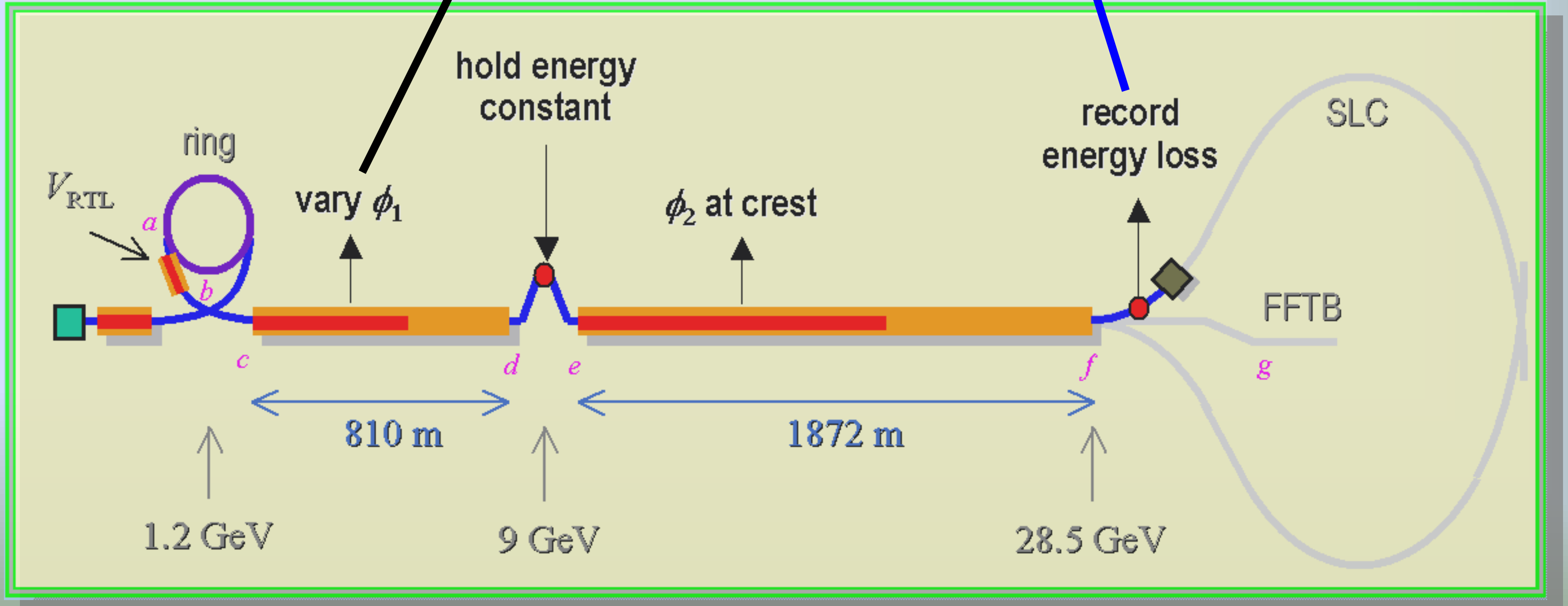
compression by factor of 500

P. Emma *et al.*, PAC'01

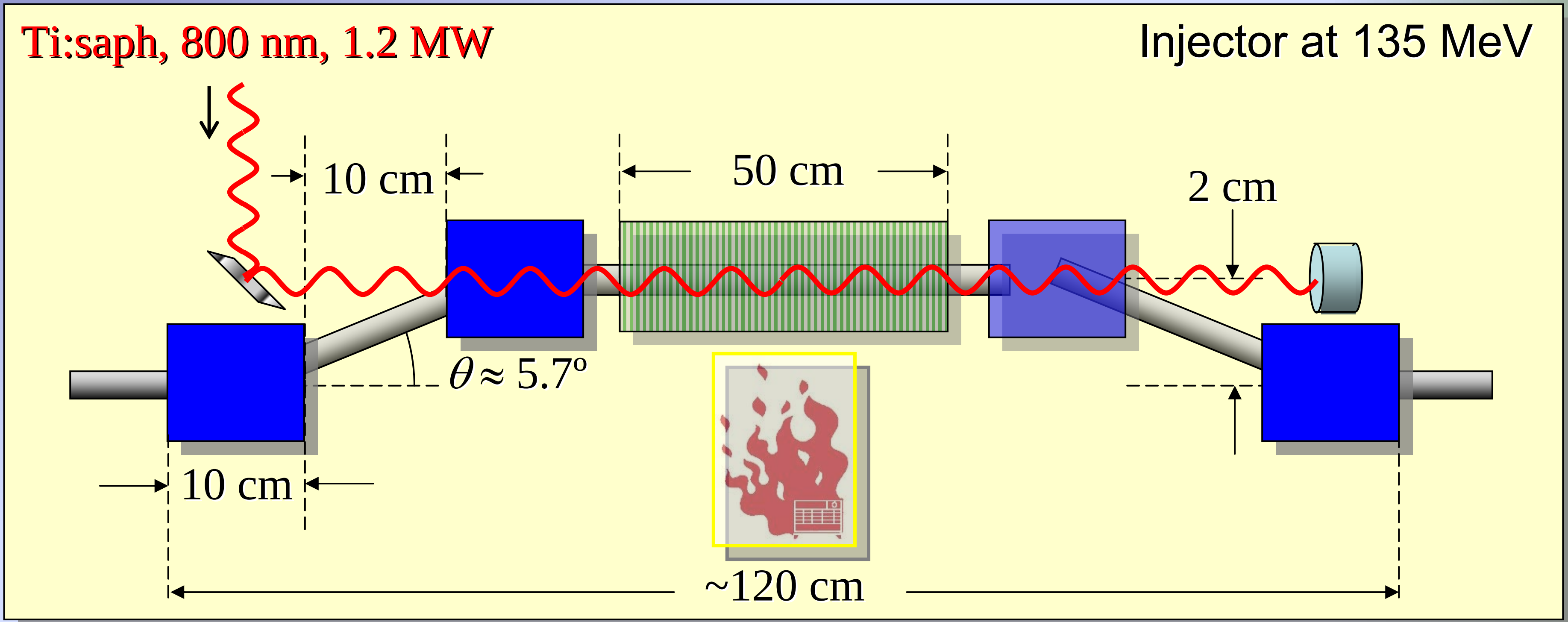
Wakefield energy-loss used to set and confirm minimum bunch length

K. Bane *et al.*, PAC'03





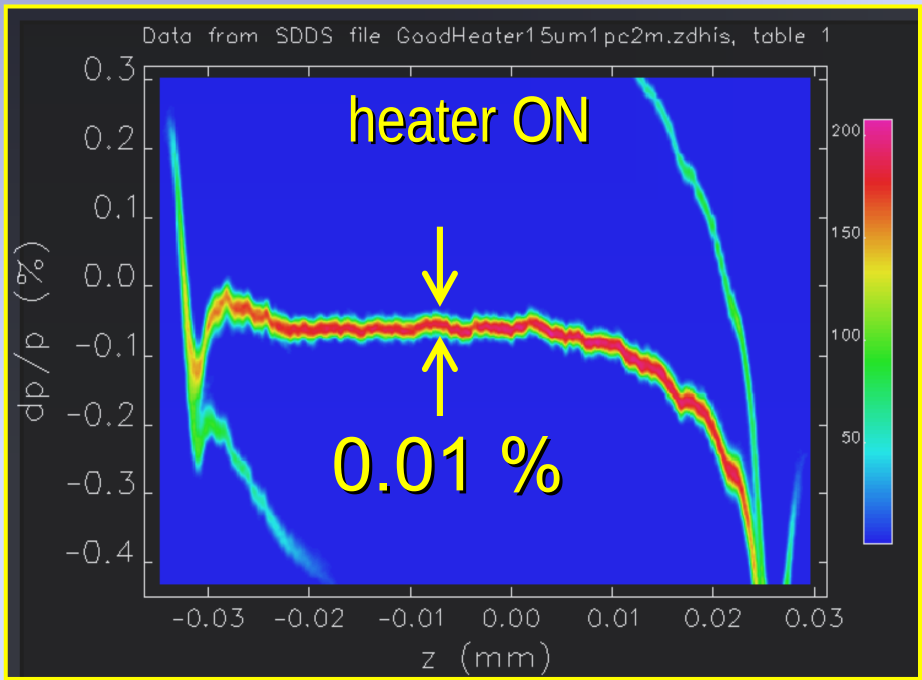
‘Laser Heater’ in LCLS for Landau Damping

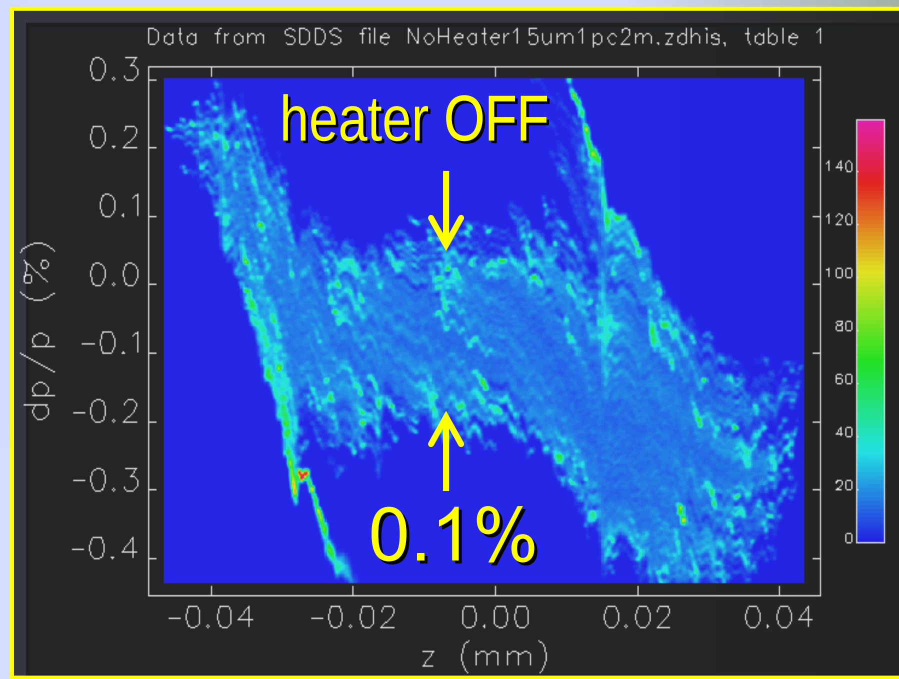


- Laser-e^- interaction induces 800-nm energy modulation $\Rightarrow$ 40 keV rms
- Heater in weak chicane for time-coordinate smearing
- Energy spread in next compressors smears μ-bunching

Huang: ***WEPLT156***, Limborg: ***TUPLT162***, Carr: ***MOPKF083***

In *LCLS* tracking, final energy spread blows up without 'Laser-Heater'

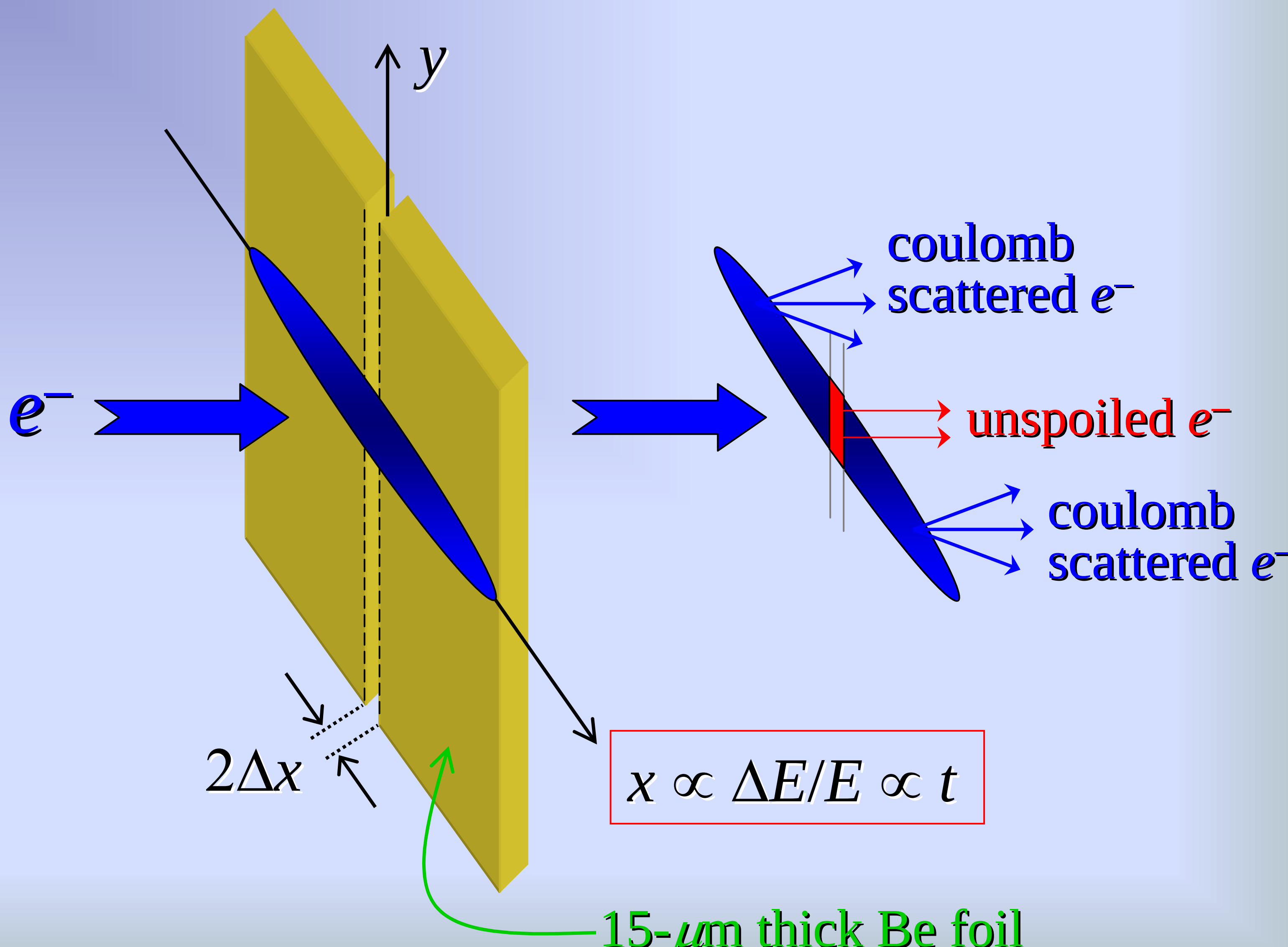




Final longitudinal phase space at 14 GeV for initial 15-μm, 1% modulation at 135 MeV

Z. Huang et al., SLAC-PUB-10334, June 2004 ...submitted to ***PR ST AB.***

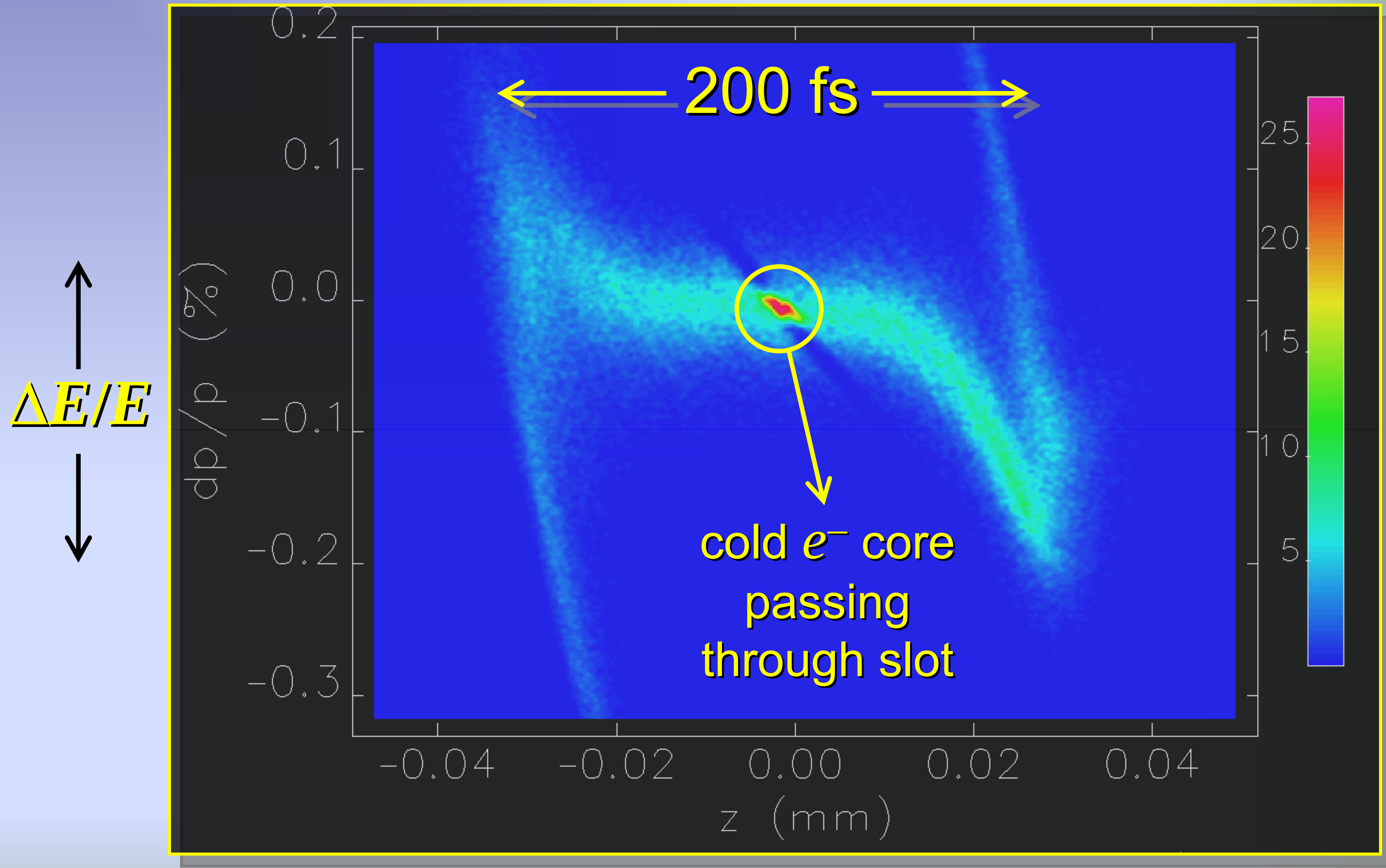
Add thin slotted foil in center of chicane



PRL **92**, 074801 (2004).

P. Emma, M. Cornacchia, K. Bane, Z. Huang, H. Schlarb, G. Stupakov, D. Walz (SLAC)

Track 200k macro-particles through entire *LCLS* up to 14.3 GeV



No design changes to FEL – only foil added in chicane