Tracking with the collimator wake fields through the CLIC-BDS

G. Rumolo, A. Latina, W. Bruns, CLIC meeting (19.05.2006)
*Thanks to J. Resta, D. Schulte, R. Tomás and F. Zimmermann

- Introduction to wake fields in the BDS
- Kicks from collimator wake fields in different regimes:
  - Geometric: diffractive, inductive, intermediate
  - Resistive wall: long-, short-range, and intermediate, w and w/o ac conductivity
- Description of a newly constructed module for the calculation of wake fields kicks to be used in tracking.
- Implementation in PLACET and first examples of PLACET tracking along the BDS including the wake fields of some flat collimators ($\rightarrow$ A. Latina)

Main contributions to the wake fields in the Beam Delivery System

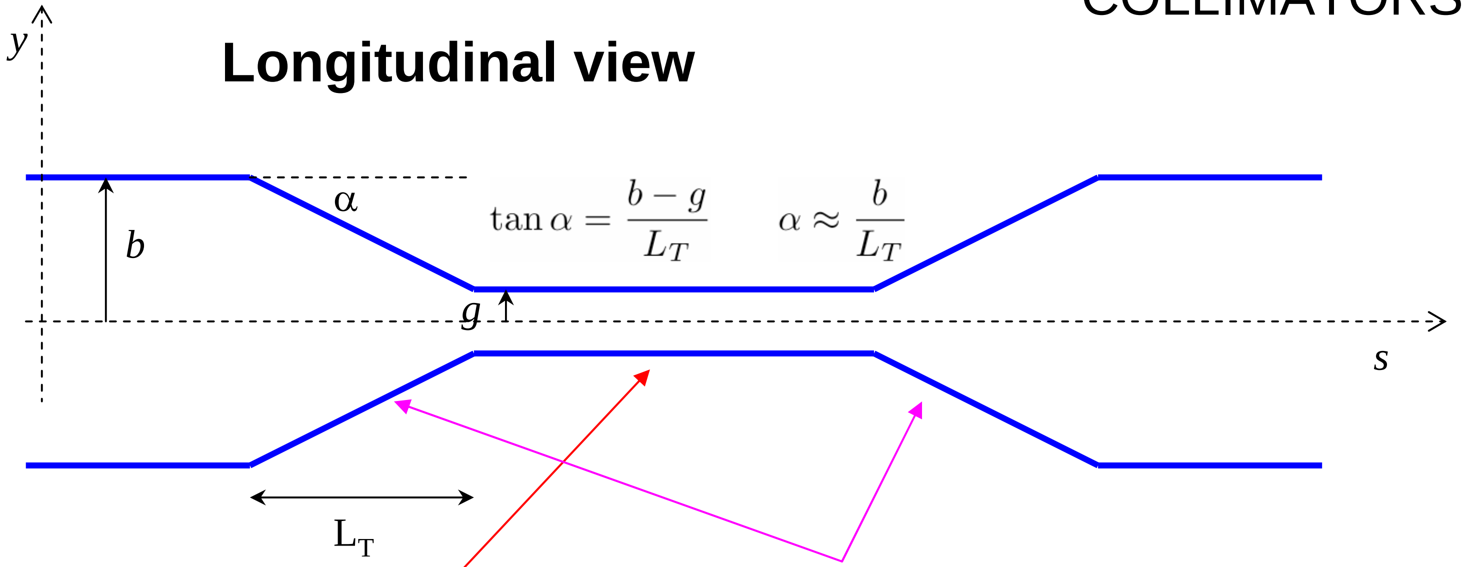
- Geometric and resistive wall wake fields of the collimators (tapered and flat parts)
- Resistive wall wakes of the beam pipe, especially close to the IP (final quadrupoles)
- Crab cavities LOM‘s and HOM‘s

⇩

Wake fields can be responsible for severe single- and multi-bunch effects leading to **luminosity loss**

COLLIMATORS & BEAM

Longitudinal view



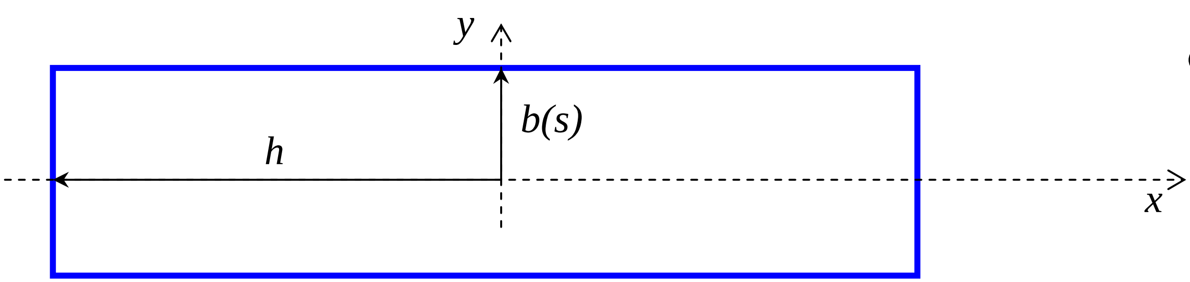
Tapered parts contribute to:

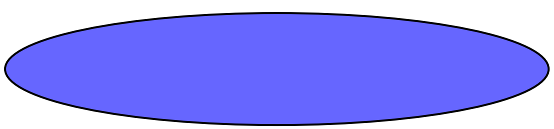
1. Geometric wakes
2. Resistive wall wakes

Flat part contribute to:

1. Resistive wall wakes

Cross-sectional view





N part/bunch

σ_z rms-bunch length

γ relativistic factor

In the following a flat geometry will be assumed in the transverse plane: ***b(s)<<h***

Geometric wake

For smooth tapering, the kick is given by *(Stupakov, 1997)*

$$\Delta y' = \frac{N r_e}{\gamma\sqrt{2\pi}\sigma_z}\left[(2\pi h I_2 - 2I_1)\Delta y + 2I_1 y\right]\exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

$$I_1 = \int_0^{L_T}\left(\frac{b'^2}{b^2}\right)ds \qquad I_2 = \int_0^{L_T}\left(\frac{b'^2}{b^3}\right)ds$$

$\rightarrow$ *(x,y,z)* are the coordinates of the particle that feels the wake force

$\rightarrow$ *Δy* is the vertical diplacement of the bunch.

$$\boxed{\Delta y' = \frac{N r_e}{\gamma\sqrt{2\pi}\sigma_z}\left[\left(\pi h\frac{(b-g)^2(b+g)}{g^2 b^2 L_T} - 2\frac{(b-g)^2}{g b L_T}\right)\Delta y + 2\frac{(b-g)^2}{g b L_T} y\right]\exp\left(-\frac{z^2}{2\sigma_z^2}\right)}$$

The old formula was $$\Delta y' = \frac{N r_e}{\gamma\sqrt{2\pi}\sigma_z}\frac{(b-g)^2}{g b L_T}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)(0.85\Delta y + 0.43 y)$$

This formula is only true in the inductive regime, which is defined by *(G. V. Stupakov, 2001):*

$$\alpha \ll \frac{g\sigma_z}{h^2}$$

|n diffraction regime $\alpha \gg \frac{g\sigma_z}{h^2}$

$$\Delta y' = \frac{N r_e \sqrt{2}}{\gamma g^2} \exp\left(-\frac{z^2}{2\sigma_z^2}\right)(0.85\Delta y + 0.43 y)$$

whereas in the intermediate regime, the following formula holds:

$$\Delta y' = \frac{2.7 N r_e \sqrt{2\alpha}}{\gamma \sqrt{\sigma_z g}} \exp\left(-\frac{z^2}{2\sigma_z^2}\right)(0.85\Delta y + 0.43 y)$$

Purpose of the **Gdfidl** simulations:

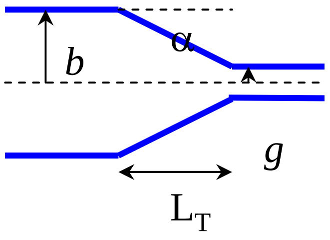
$\Rightarrow$ Check the analytical formulae known from literature (Stupakov) about geometric wake fields

Gdfidl simulations are done by offsetting by $\Delta y=20\mu m$ a short Gaussian pulse ($\sigma_z=100\mu m$) with a $1pC$ charge through a taper and calculating the resulting wake potential *w(s)* (defined as the integrated electromagnetic force felt by a witness unitary charge at a distance *s* from the source).

$\rightarrow$ The taper geometry is specified in the following:

h scanned from 1mm to 5mm (with step 1mm)

L_T =25mm, b=0.8mm, g=0.1mm



With the parameters used in these simulations it turns out that we are in the so-called „intermediate“ regime -> not very smooth tapering

The condition for smooth tapering is given by:

$$\alpha \ll \frac{g\sigma_z}{h^2}$$

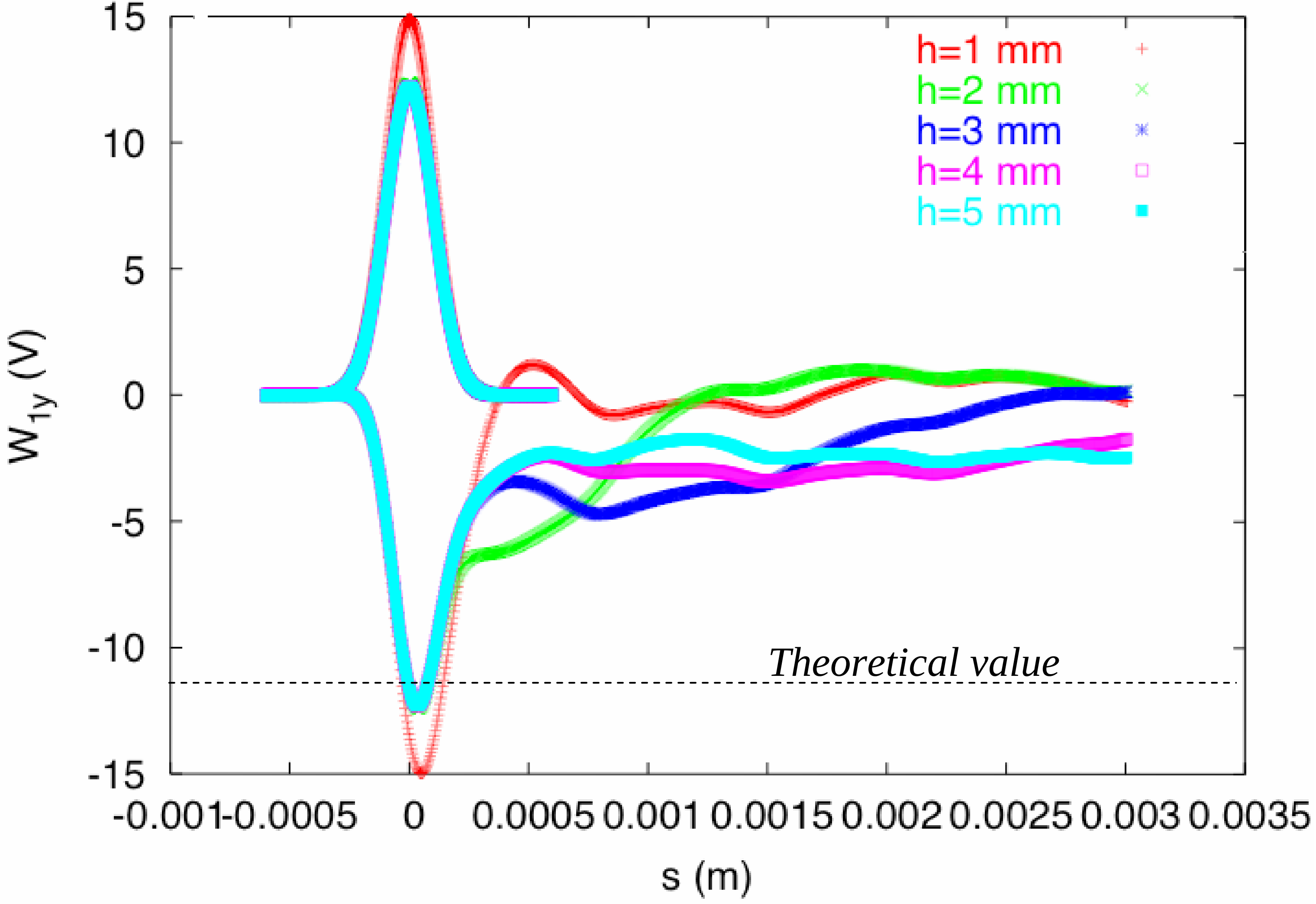
α is the tapering angle

$$\frac{g\sigma_z}{h^2} = 0.08 \cdot \frac{1}{h^2[\mathrm{mm}^2]}$$

$$\alpha = \arctan\left(\frac{b-g}{L_T}\right) = 0.028$$

h should be smaller than 1 mm to meet the condition of inductive regime. Our simulations have been run for h=1 to 5 mm, therefore we will be in intermediate regime.

$$w(s) = \frac{Z_0 c}{4\pi}\frac{2.7\sqrt{2\alpha}Ne}{\sqrt{\sigma_z g^3}}\Delta y \exp\left(-\frac{s^2}{2\sigma_z^2}\right) \quad \Rightarrow \quad w_{max} = \frac{Z_0 c}{4\pi}\frac{2.7\sqrt{2\alpha}Ne}{\sqrt{\sigma_z g^3}}\Delta y = 11.5\mathrm{V}$$



Results from W. Bruns‘ Gdfidl simulations.

The upper curve represents the probe bunch (normalized to the highest value of the wake for plotting purposes) and the lower curves are the wakes referring to the labelled cases.

Conclusions that can be drawn from W. Bruns‘ simulations and work yet to be done (..underway)

• As expected, the wake field in the intermediate regime does not depend (strongly) on *h*. The predicted maximum value from the analytical formula matches quite well the results of the Gdfidl simulations.

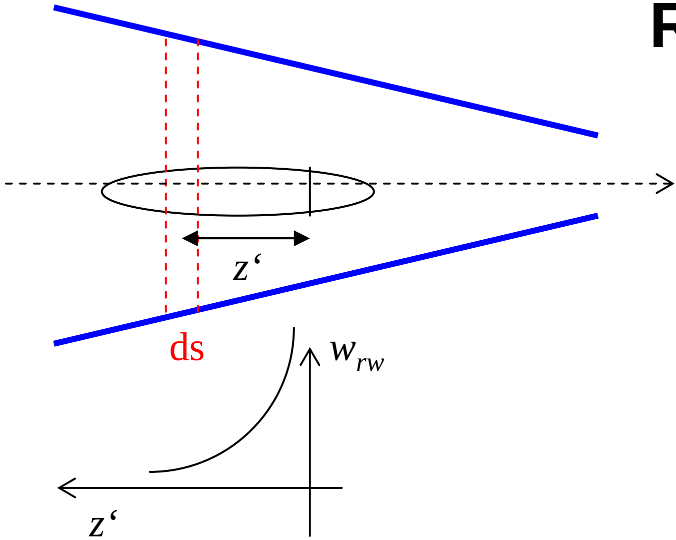
• From the simulations it appears though that there is a „trailing effect“ at the bunch tail that seems to depend on *h*. For higher values of *h*, the wake does not vanish after the bunch passage, which could matter for multi-bunch effects.

• More simulations are being run to check the analytical formulae in the inductive regime (we are specially interested to cross-check the predicted dependence on *h* of the wake). We could for instance:

$\rightarrow$ Increase the taper length by a factor 10 (unfortunately not feasible because the computing time would become too large)

$\rightarrow$ Simulate a bunch 10 times longer (what we are doing)

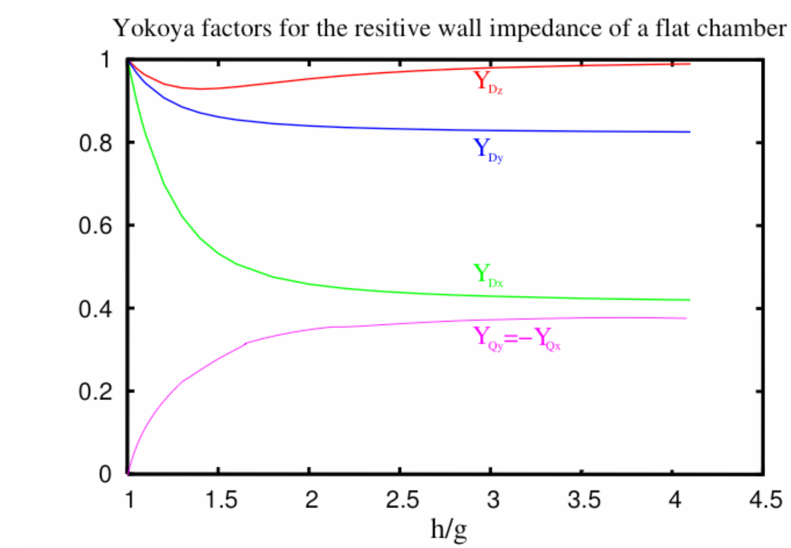
Resistive wall wake (long range)



The contribution to the kick of a particle at the longitudinal position *z* from the piece *ds* of taper is given by (neglecting possible centroid position variations along the bunch):

$$\Delta y'(s) = \frac{4Nr_e\sqrt{\lambda}ds}{\sqrt{2}\pi\gamma\sigma_z b^3(s)}\left[Y_{Dy}(s)\Delta y + Y_{Qy}(s)y\right]\int_0^\infty \frac{1}{\sqrt{z'}}\exp\left[-\frac{(z+z')^2}{2\sigma_z^2}\right]dz'$$

$Y_{Dy}(s)$ and $Y_{Qy}(s)$ are the Yokoya factors associated to dipole and quadrupole wake fields in a flat chamber, $\lambda = (c\sigma\mu_0)^{-1}$



if $\dfrac{h-b(s)}{h+b(s)} \geq 0.4$ we can assume $\begin{aligned} Y_{Dy}(s) &\approx \pi^2/12 \\ Y_{Qy}(s) &\approx \pi^2/24 \end{aligned}$

and we can analytically carry out the integration in *ds* over the whole taper length, so we obtain

$$\Delta y' = \frac{4Nr_e\sqrt{\lambda}(b+g)L_T}{2\sqrt{2}\pi\gamma\sigma_z b^2 g^2} \int \frac{1}{\sqrt{z'}} \exp\left[-\frac{(z+z')^2}{2\sigma_z^2}\right] dz' \left(Y_{Dy}\Delta y + Y_{Qy}y\right)$$

whereas for the flat part it simply holds:

$$\Delta y' = \frac{Nr_e\sqrt{\lambda}L_f}{\gamma\sqrt{2\pi}\sigma_z g^3} \int_0^\infty \frac{1}{\sqrt{z'}} \exp\left[-\frac{(z+z')^2}{2\sigma_z^2}\right] dz' \left[Y_{Dy}\Delta y + Y_{Qy}y\right]$$

This resistive wall wake fields are not applicable in all cases but only if...

1) Penetration skin depth is much smaller than the pipe transverse size, which translates into wake distances such that

$$\delta_{skin}\left(\frac{c}{z}\right) << g \Rightarrow |z| << \frac{2g^2}{\lambda}$$

2) In the formula for the resistive wall impedance below we can neglect the second term in the sum at the denominator (*m=1*)

$$\frac{Z_m^{||}(\omega)}{L} = \frac{\omega}{c}\frac{Z_m^{\perp}(\omega)}{L} = \frac{cZ_0}{\pi}\frac{4/g^{2m}}{(1+\delta_{m0})gc\sqrt{\frac{cZ_0\sigma}{2|\omega|}}(1+\mathrm{sgn}(\omega)) - \frac{ig^2\omega}{m+1}}$$

$$\frac{g^2\omega}{2} << gc\sqrt{\frac{cZ_0\sigma}{2|\omega|}} \Rightarrow |z| >> 0.63 s_0 \qquad \text{with} \qquad s_0 = \left(\frac{2g^2}{Z_0\sigma}\right)^{1/3}$$

Resistive wall wake (intermediate and short range w or w/o a.c. conductivity) *(K. Bane & M. Sands,'95)*

$$W_1^{\perp}(z,s) = \frac{cZ_0}{\pi b^3(s)}\left[\frac{s_0 \exp\left(-\alpha_t \frac{z}{s_0}\right)}{3(\alpha_t^2+k_t^2)}\left(\alpha_t \cos(k_t \frac{z}{s_0}) + k_t \sin(k_t \frac{z}{s_0}) - \alpha_t\right) - \right.$$

$$\left. -\frac{\sqrt{2}}{\pi}\int_0^z \int_0^\infty \frac{x^2 \exp\left(-x^2 \frac{z'}{s_0}\right) dx}{x^6+8} dz'\right]$$

→ but careful because $s_0(s)$, $\alpha_t(s)$, $k_t(s)$!!!!

$$\Delta y'(s) = \frac{N r_e ds}{\gamma\sqrt{2\pi}\sigma_z}\int_0^\infty W_1^{\perp}(z',s)\exp\left[\frac{(z'+z)^2}{2\sigma_z^2}\right] dz' \cdot$$

$$\cdot(0.85\Delta y + 0.43 y)$$

$$\Delta y' = \int_{L_c} \Delta y'(s) ds$$

Resistive wall wake (...)

→ In the intermediate regime the full formula for the kick needs to be evaluated

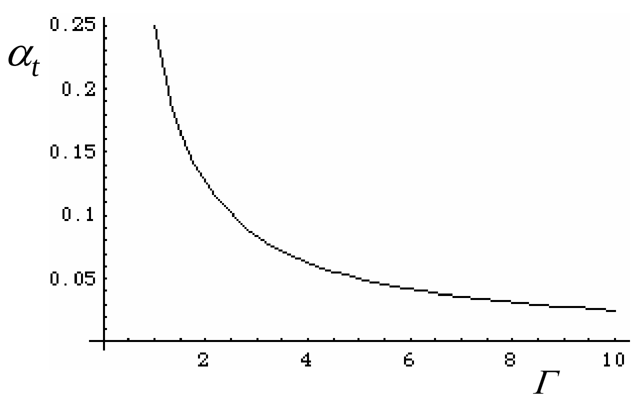
→ In the short-range regime, the integral in the $W_1(z,s)$ can be dropped and only the broad-band resonator part is left.

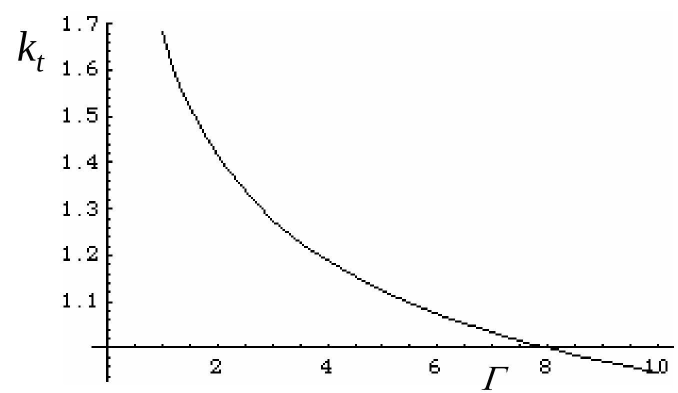
→ In the dc-conductivity regime α_t=1 and k_t=1.7

→ In the ac-conductivity regime: $\boxed{\sigma(\omega) = \frac{\sigma_{dc}}{1 - i\omega\tau}}$

α_t and k_t are functions of the relaxation factor $\Gamma = c\tau/s_0$.

Γ>1





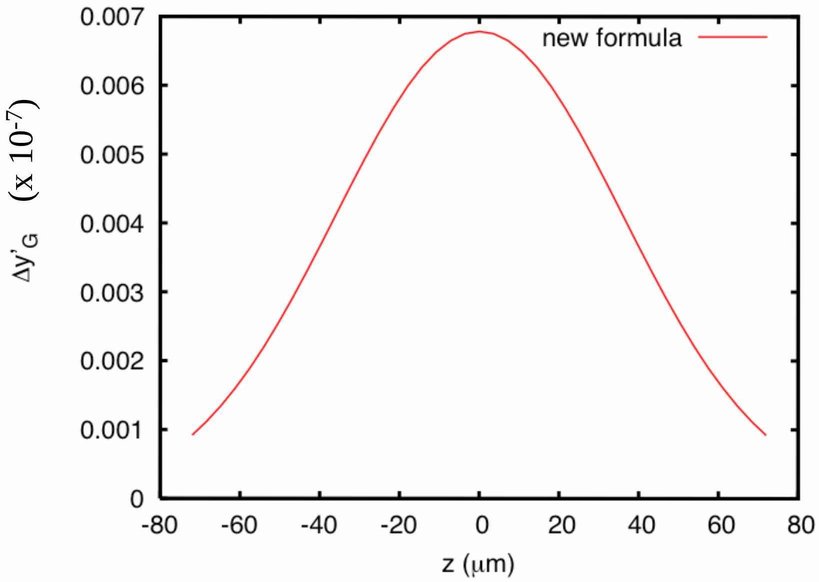
Examples of calculation of the collimator wake fields

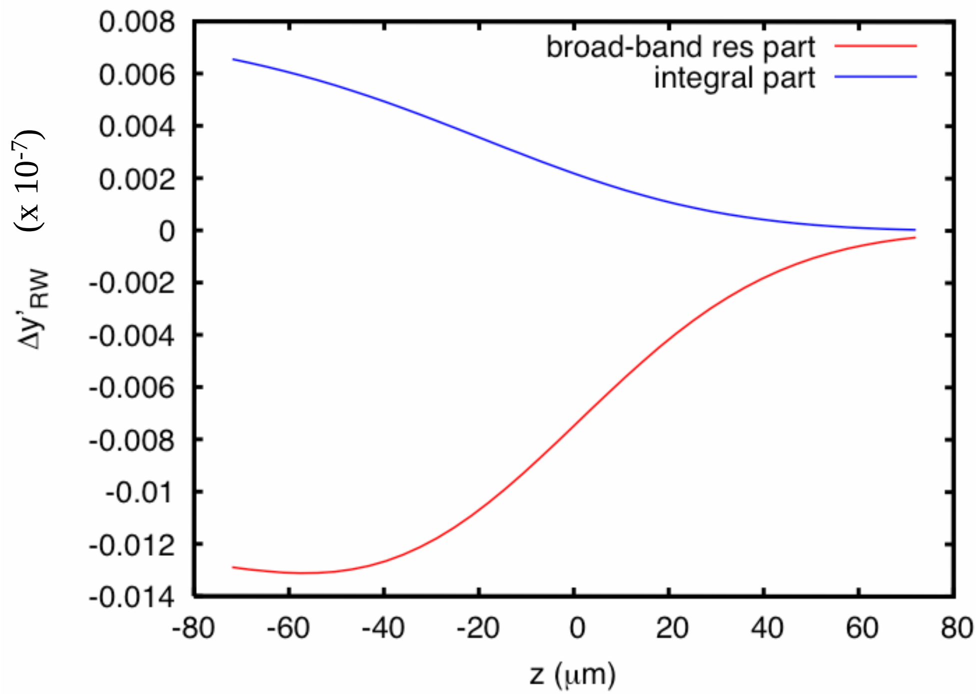
- A module for the calculation of the collimator wake fields in different regimes has been constructed to be implemented in the PLACET tracking code
- Based on the parameters (‚beam‘ and ‚collimator‘ structures), the module first determines *the type of regime* (geometric and resistive), then evaluates the kick as function of the longitudinal position, and applies it to the bunch particles accordingly.
- Results from testing in different regimes are shown in the following slides.

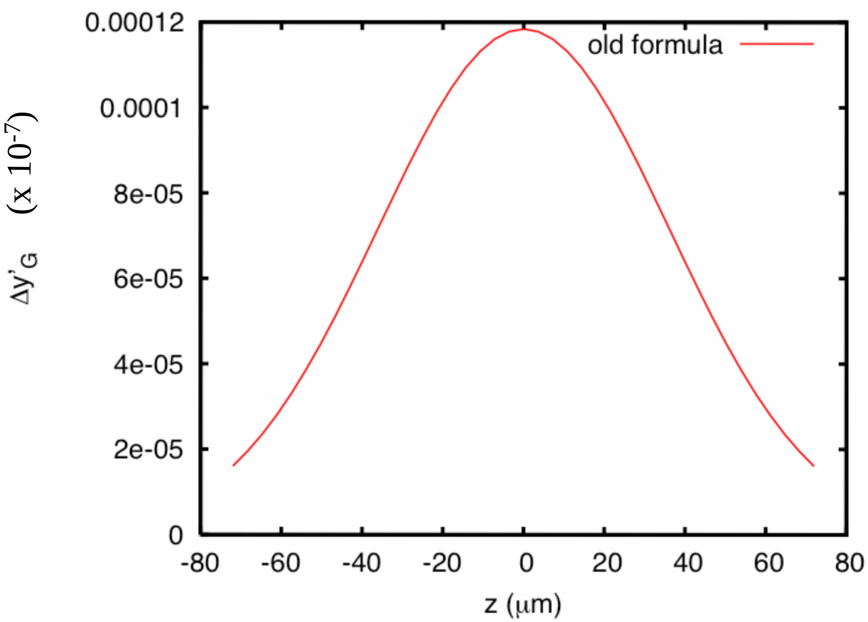
```
Nb = 5.6e9;           /* number of electrons in one bunch           */
sz0 = 36.e-6;         /* rms-length of the bunch
                         CLIC intermediate range                    */
//sz0 = 3.6e-7;       /* rms-length of the bunch
                         short range                                */
//sz0 = 8.3e-4;       /* rms-length of the bunch
                         long range                                 */
gam = 3.e6;           /* relativistic gamma                         */
bb = 8.025e-4;        /* pipe initial height (m)                    */
gg = 1.e-4;           /* pipe final height (m)                      */
ww= 2.e-3;            /* pipe width (m)                             */
LT = 0.5e-2;          /* taper length  (m)                          */
Lflat = 3.e-2;        /* length of the flat part (m)                */
sig_ch = 6.e4;        /* Conductivity of the collimator material
                         [Ohm m]^(-1)  carbon                       */
//sig_ch = 1.02e6;    /* Conductivity of the collimator material
                         [Ohm m]^(-1)  copper                       */
tau_ch=1.e-15;        /* relaxation time of the collimator material */

coll=(COLLIMATOR*)xmalloc(sizeof(COLLIMATOR));
coll->in_height=bb;
coll->fin_height=gg;
coll->width=ww;
coll->taper_length=LT;
coll->flat_length=Lflat;
coll->sigma=sig_ch;
coll->tau=tau_ch;
```

Intermediate range: geometric and resistive wall





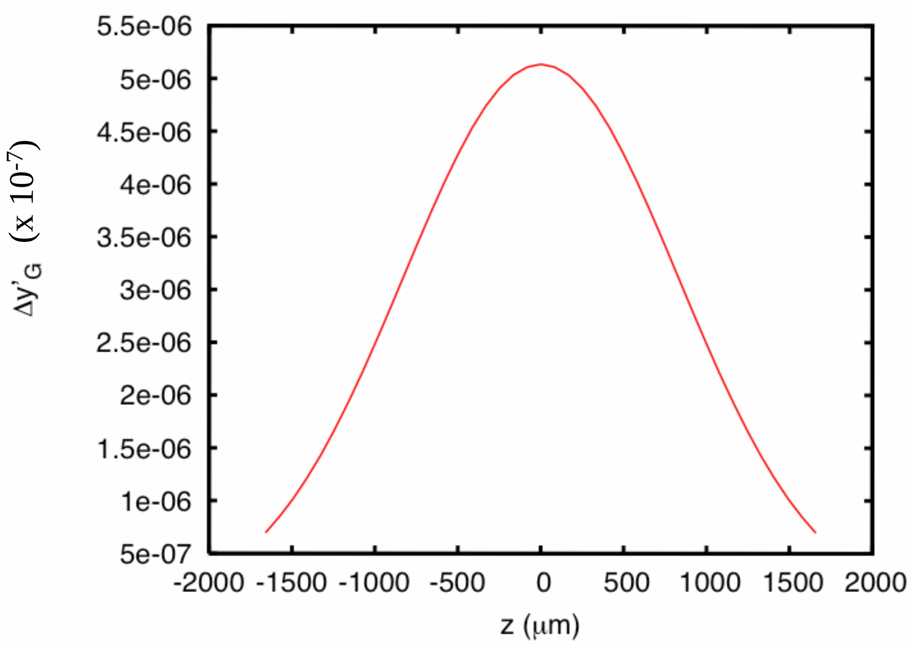


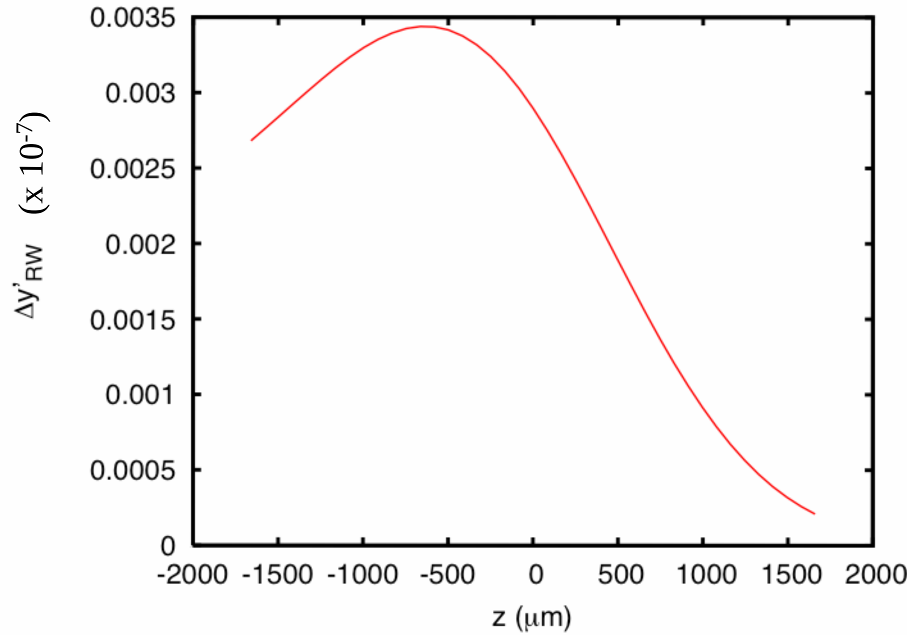
Geometric part: shown both with the old and the new formula

Resistive wall part: the two contributions to the kick are shown separately

The two contributions (geometric and resistive) are the same order of magnitude.

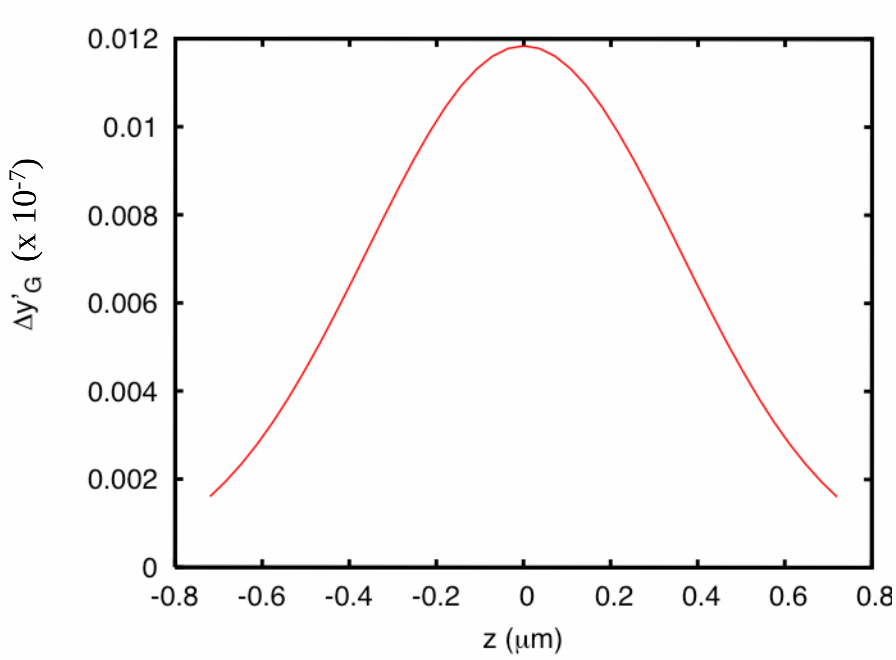
Long range: geometric and resistive wall

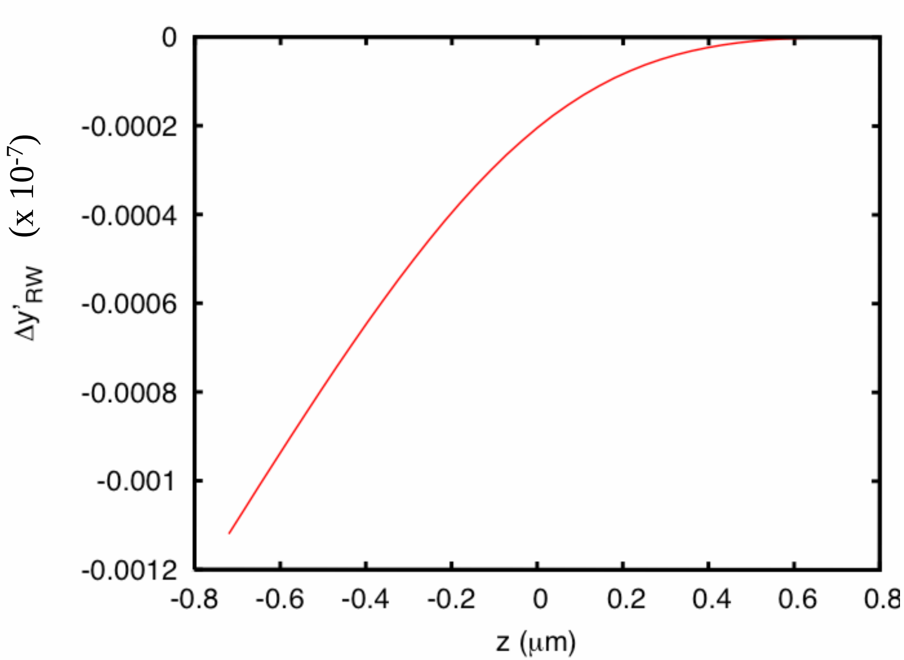




We have considered a longer bunch to artificially be in long range regime and test the code. The geometric part seems to be in this regime much smaller than the resistive wall contribution

Short range: geometric and resistive wall



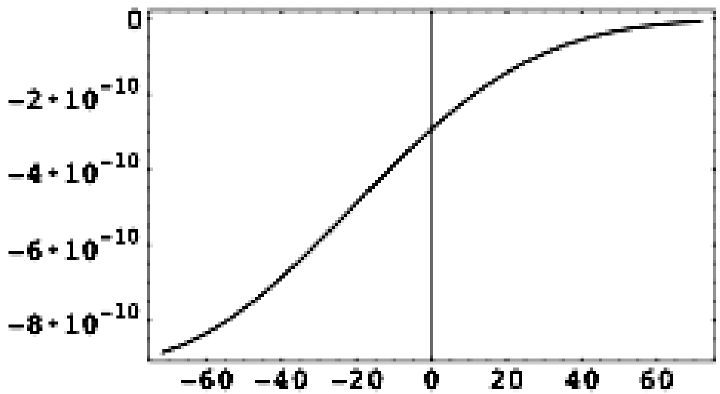


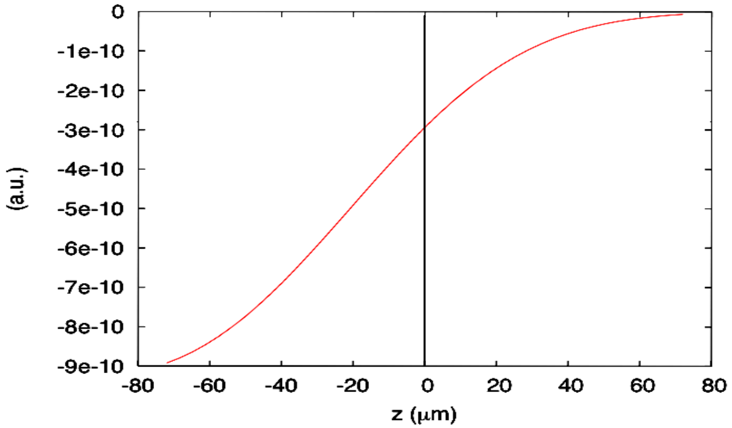
We have considered a shorter bunch to artificially be in short range regime and test the code. The geometric part seems to be in this regime larger than the resistive wall contribution

TEST: Comparison of calculation of the intermediate range wake function...

$$W_1^{\perp}(z,s) = \frac{cZ_0}{\pi b^3(s)}\left[\frac{s_0 \exp\left(-\alpha_t \frac{z}{s_0}\right)}{3(\alpha_t^2+k_t^2)}\left(\alpha_t \cos(k_t\frac{z}{s_0}) + k_t \sin(k_t\frac{z}{s_0}) - \alpha_t\right) - \frac{\sqrt{2}}{\pi}\int_0^z \int_0^\infty \frac{x^2 \exp\left(-x^2 \frac{z'}{s_0}\right) dx}{x^6+8} dz'\right]$$

... in $s=0$ and over the bunch using some typical CLIC numbers...





... with MATHEMATICA or...   ... with our module

Partial conclusions

- A C-module for wake fields has been constructed to allow tracking with collimator wake fields according to the regime (both geometric and resistive wall)
- Testing in all regimes (both geometric and resistive wall) has been carried out and the resulting wake fields have been cross-checked in a few selected cases with MATHEMATICA obtaining an excellent agreement.
- More to be described about the implementation in PLACET of the module $\Rightarrow$ **the story continues with Andrea‘s talk....**