Cockcroft

Institute

Daresbury

THE COCKCROFT INSTITUTE *of*
ACCELERATOR SCIENCE and TECHNOLOGY

http://www.lancs.ac.uk/cockcroft-institute/

Sir John Cockcroft FRS

b. Todmorden (Lancashire *and* Yorkshire!)

ed. Manchester University: Maths

Manchester College of Technology (UMIST): Elec. Eng.

Metropolitan-Vickers, Manchester

PhD then post-doc, Cambridge Univ.

Nobel Laureate, Physics, 1951

CCLRC

LANCASTER UNIVERSITY
Celebrating 40 Years of Excellence

THE UNIVERSITY of LIVERPOOL

MANCHESTER 1824
The University of Manchester

Northwest
REGIONAL DEVELOPMENT AGENCY

PPARC

History

- why here in NW England ?
  - Daresbury ↔ accelerator-lead research univs

    Liverpool Lancaster Manchester

    Nuclear Physics (since Rutherford !)

    High Energy Physics (since Chadwick !)

    Synchrotron Radiation science (since SRF 1970s)
  - all require new accelerator systems for progress
    - all have been on Daresbury campus in their time
- Cockcroft/Walton experience 70 years on

> "… they were fortunate to have the support of Metropolitan Vickers: … the Manchester company."
>
> B Cathcart in "The Fly in the Cathedral"

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R&D Challenge

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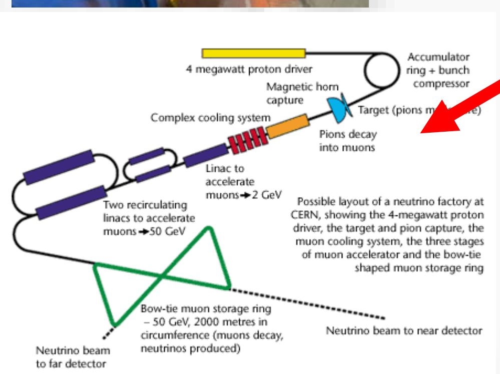
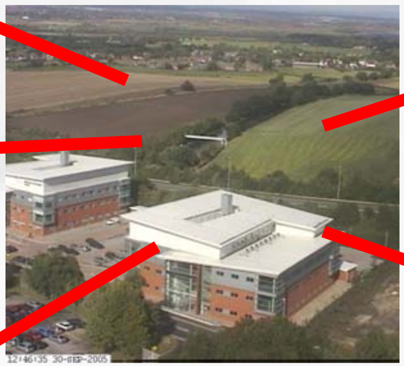
High Energy Physics

- global
- universal
  - MV/m
  - intensity
  - nm delivery

"light"



ν-Factory



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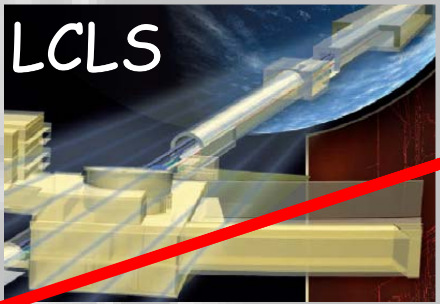

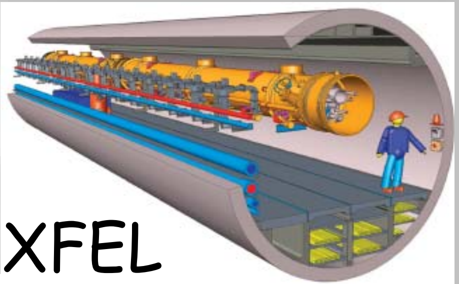

XFEL

neutrons

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RC-UK Facilities

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• UK Funding Sources

Large Facility	07/08	08/09	09/10	10/11	11/12	12/13	13/14	14/15	15/16	16/17	17/18	18/19	19/20
Supermemo (PPARC)													
Upgrade the Mega Amp Spherical Tokamak (MAST) at Culham (EPSRC)													
Household Panel Study (ESRC)													
New Scientific Opportunities at the European Synchotron Radiation Facility (CCLRC) A							EPSRC						
4GLS (CCLRC) A							EPSRC						
UK Participation in the construction of a facility for antiproton and ion research (EPSRC) A							EPSRC						
Oceanographic Research Ship (NERC)													
National Institute for Medical Research (NIMR) (MRC)													
ISIS Second Target Station Instruments (CCLRC) !													
The European X-Ray Laser Project (CCLRC) A									EPSRC				
Linear Collider (PPARC) A									PPARC				
Gravitational Wave Detection Facilities (PPARC)													
A Megawatt Class Spallation Neutron Source for Europe (CCLRC) ! A									EPSRC				
Extremely Large Telescope (ELT) (PPARC)													
European High Performance Computing Service (EPSRC)													
Diamond Phase III (CCLRC) A										EPSRC			
Neutrino Factory (PPARC) A										PPARC			
HIPER: High Power Experimental Research facility (CCLRC)													
Mini Fabrication facility for Nanotechnology (EPSRC)													
Square Kilometre Array (PPARC)													

Key: £0-10m | £10-25m | £25-50m | £50m+

! SNS (1 MW) from 2007

! JPARC (1 MW) from 2009/10 ?

EPSRC science

PPARC science

A accelerator science and technology

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Accelerators Today

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- accelerators today drive wealth creation
  - accelerator technology of the 20^{th} Century
  - from the physics of the 20^{th} Century

General industrial use:
Sterilisation, imaging

Research accelerators:
Particles, synchrotron light used in biomedical, physics, chemistry, biology, material research

Radiotherapy:
Cancer treatment with X-rays, protons and other particles

Ion implantation, surface modifications:
Controlled semiconductor doping; Changing properties of surfaces

Radioisotope production:
Cancer treatment; imaging organs for medical use

Cockcroft Institute +UK plc

- accelerators tomorrow ?
  - accelerator science ↔ KT ↔ UK plc

Mission

The Institute's "mission" is summarised in the following "deliverables":

- generic R&D in Accelerator Science and Technology (AST);
- project specific R&D in AST
  (e.g. a linear collider and a Neutrino Factory);
- leadership and management of national deliverables to international facilities (which may be UK-situated);
- competence in crucial and specific technologies;
- technology transfer to industry;
- staff complement of internationally acknowledged expertise;
- seamless involvement of the HEI and CCLRC sectors;
- education and training to ensure a flourishing staff supply side.

Participation

- unique, seamless, collaboration of established research excellence in NW England

"faculty" + pdocs + students
now 30 FTE on to 60 FTE

senior + junior scientists
+ students now 50 FTE

- seamless synergy of basic science
  engineering and technology
  industry

↳ UK inward investment
  - science → people, skills, innovation, progress
  - high-tech → industrial production
  - economic development (national + regional)

New mathematical modelling of ultra-relativistic charge

David Burton
Jonathan Gratus
Robin Tucker

Lancaster University and the Cockcroft Institute

Charged Continua with Self-Fields

- A new approach for analysing the dynamic behaviour of distributions of charged particles in an electromagnetic field

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- Employs intrinsic tensor analysis and exploits the symmetries and light-cone structure of spacetime

Charged Continua with Self-Fields

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- Yields hierarchy of mainly linear equations for an asymptotic approximation for self-consistent radiation fields and charged currents in ultra-relativistic configurations
- Employs intrinsic tensor analysis and exploits the symmetries and light-cone structure of spacetime
- Offers a powerful and systematic method for the analysis of coherent radiation from collections of charge in complex accelerating devices

Lorentz-Dirac Equation for a Point Charge

In the presence of an external Maxwell field $\mathcal{F}_{\text{ext}}$ the motion of a point charge based on a particular mass-renormalisation contains in addition to the Lorentz force, $q_0 i_V \mathcal{F}_{\text{ext}}$, a radiation reaction force proportional to the proper rate of change of the particle's $4-$acceleration $\dot{\mathcal{A}}$:

$$m_0 c^2 \mathcal{A} = q_0 i_V \mathcal{F}_{\text{ext}} + \frac{2}{3} \frac{q_0^2}{4\pi\epsilon_0} \Pi_V \dot{\mathcal{A}}$$

where m_0 is the rest mass of the particle with electric charge q_0.

Although solutions to this system that self-accelerate can be eliminated by demanding contrived data at different points along the world-line there remain solutions that pre-accelerate in situations where the external field is piecewise defined in spacetime.

Landau-Lifshitz Reduction

One resolution of these difficulties is to assume that the right hand side of the equation should be expanded as a series in q_0 with leading term for $\tilde{\mathcal{A}}$ given by $\frac{q_0}{mc^2} i_V \mathcal{F}_{\text{ext}}$. Then to some order in q_0

$$\mathcal{A} = \frac{q_0}{m_0 c^2} i_V \mathcal{F}_{\text{ext}} - \frac{2}{3 m_0 c^2} \frac{q_0^2}{4\pi\epsilon_0} i_V (\widetilde{V} \wedge \nabla_V \mathcal{A}_{\text{ext}}) + \ldots$$

where $\mathcal{A}_{\text{ext}} = \frac{q_0}{m_0 c^2} i_V \mathcal{F}_{\text{ext}}$. The system is now manifestly a second order system of evolution equations. Although this offers a workable scheme it is unclear what its limitations are in different types of external field.

In situations where one has to contemplate the radiation from a large number of accelerating high-energy particles in close proximity the neglect of higher order terms in the expansion may be suspect.

Material Stress-Energy Tensor

- A thermodynamically inert (cold) fluid can be modelled with the material stress-energy-momentum tensor

$$T^{(\mathrm{f})} = \frac{m_0}{c\epsilon_0} \mathcal{N} \widetilde{V} \otimes \widetilde{V}$$

  where $\mathcal{N}$ is a scalar number density field, m_0 some constant with the dimensions of mass, V the unit time-like 4-velocity field of the fluid ($g(V, V) = -1$).

- This tensor can be added to the stress-energy-momentum tensor of the electromagnetic field to yield the total stress-energy-momentum tensor for the complete interacting system.

Charged Fluid Dynamics

If one assumes that the electric current 3-form is $j = q_0 \mathcal{N} \star \widetilde{V}$ for some electric charge constant q_0, symmetry vector fields K_μ and that $\mathcal{N}$ is *regular* then the conservation laws

$$d\, j = 0$$

$$d(\tau^{(\mathrm{EM})}_{K_\mu} + \tau^{(\mathrm{f})}_{K_\mu}) = 0$$

yield the field equation of motion

$$\nabla_V \widetilde{V} = \frac{q_0}{m_0 c^2} i_V \mathcal{F}.$$

This equation must be solved consistently with the Maxwell equations to determine $V, \mathcal{N}$ and $\mathcal{F}$ for prescribed initial and boundary conditions.

The Charged Fluid System

If the flow has a well defined velocity V at all times the complete set of field equations is

$$
\begin{aligned}
dF &= 0\,,\\
d\star F &= -\rho \star \widetilde{V}\,,\\
\nabla_V \widetilde{V} &= i_V F\,,\\
V\cdot V &= -1
\end{aligned}
$$

for the triple (V, ρ, F).

The Charged Fluid System

In general the state of the fluid may develop sheets of high density charge separating regions in space containing multiple electric currents J_n. The the field follows from the Maxwell system

$$dF = 0$$

$$d \star F = -\sum_{n=1}^{N} J_n$$

where the partial currents J_n are calculated in a "Lagrangian" picture in terms of a (folded one to many) map. In this approach the number N of currents in different regions of space is dynamical and depends on the initial conditions for the charge distributions.

Field Equations for a Charged Laminar Flow in terms of $\mathrm{p}, \rho, \mathrm{e}$ and b

$$\nabla \cdot \boldsymbol{e} = \frac{m_0 c^2}{q_0} \gamma \rho, \qquad \nabla \times \boldsymbol{b} = \frac{1}{q_0} \rho \boldsymbol{p} + \frac{1}{c^2} \frac{\partial \boldsymbol{e}}{\partial t},$$

$$\nabla \times \boldsymbol{e} + \frac{\partial \boldsymbol{b}}{\partial t} = 0, \qquad \nabla \cdot \boldsymbol{b} = 0,$$

$$\gamma \frac{\partial \boldsymbol{p}}{\partial t} + \left(\frac{\boldsymbol{p}}{m_0} \cdot \nabla \right) \boldsymbol{p} = q_0 \left(\gamma \boldsymbol{e} + \frac{1}{m_0} \boldsymbol{p} \times \boldsymbol{b} \right),$$

$$-\gamma^2 + \frac{\boldsymbol{p} \cdot \boldsymbol{p}}{m_0^2 c^2} = -1,$$

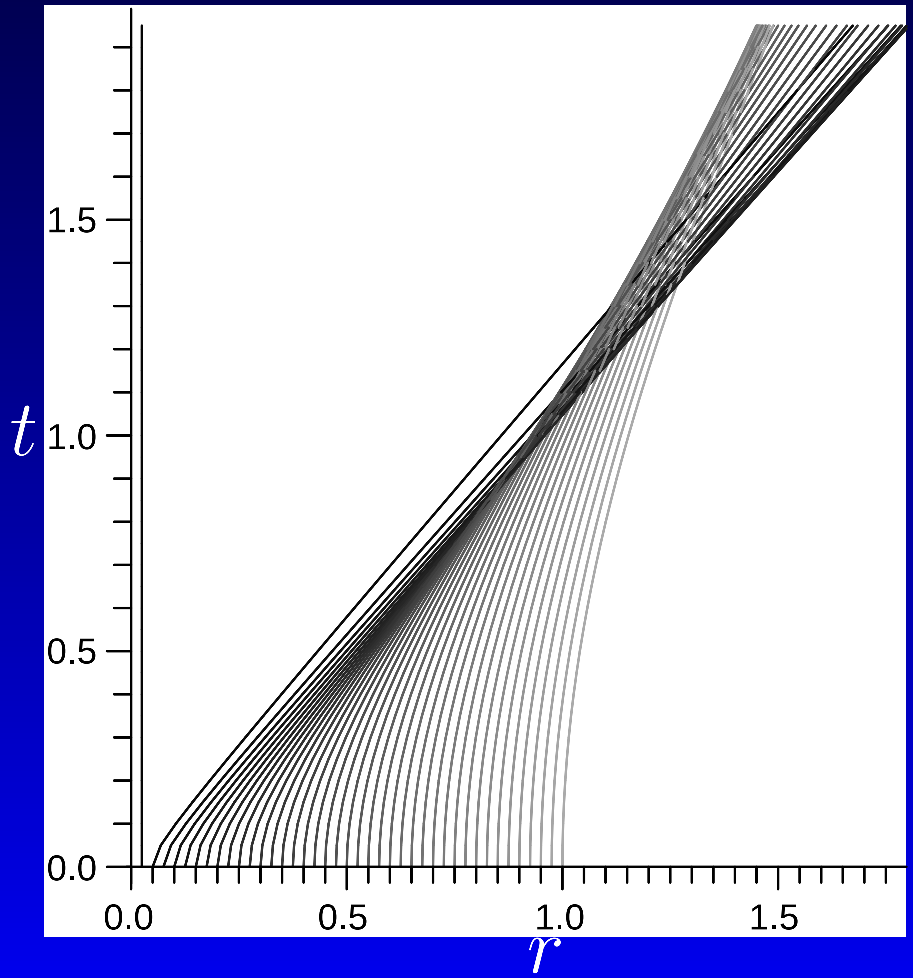
$$m_0 \frac{\partial}{\partial t} (\gamma \rho) + \nabla \cdot (\rho \boldsymbol{p}) = 0$$

Charge Density Singular Sheets

- The proper charge densities can form regions of high density reminiscent of turbulence mixing in fluid dynamics. The region outside the “fan” has only one partial current whereas the region inside the “fan” contains three partial currents.
- Each new partial current generates an additional contribution to the self-field of the charged bunched.
- The Lagrangian theory discussed here features an N-phase electric current where N is dynamically determined and has a point-wise dependence on spacetime. Configurations with $N = 1$ initially may evolve into highly complicated “turbulent” configurations where N is arbitrarily large.

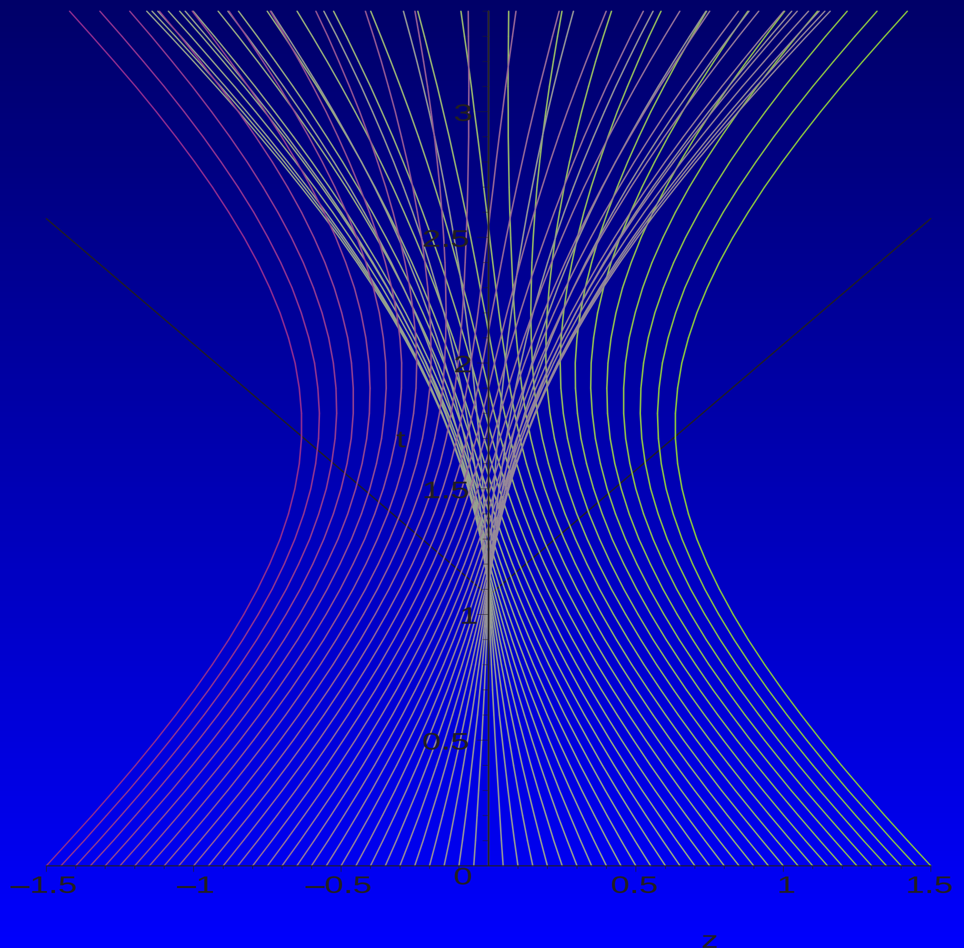
Bunch Constituent Worldlines

Spherical Gaussian Bunch expanding Under Self Forces



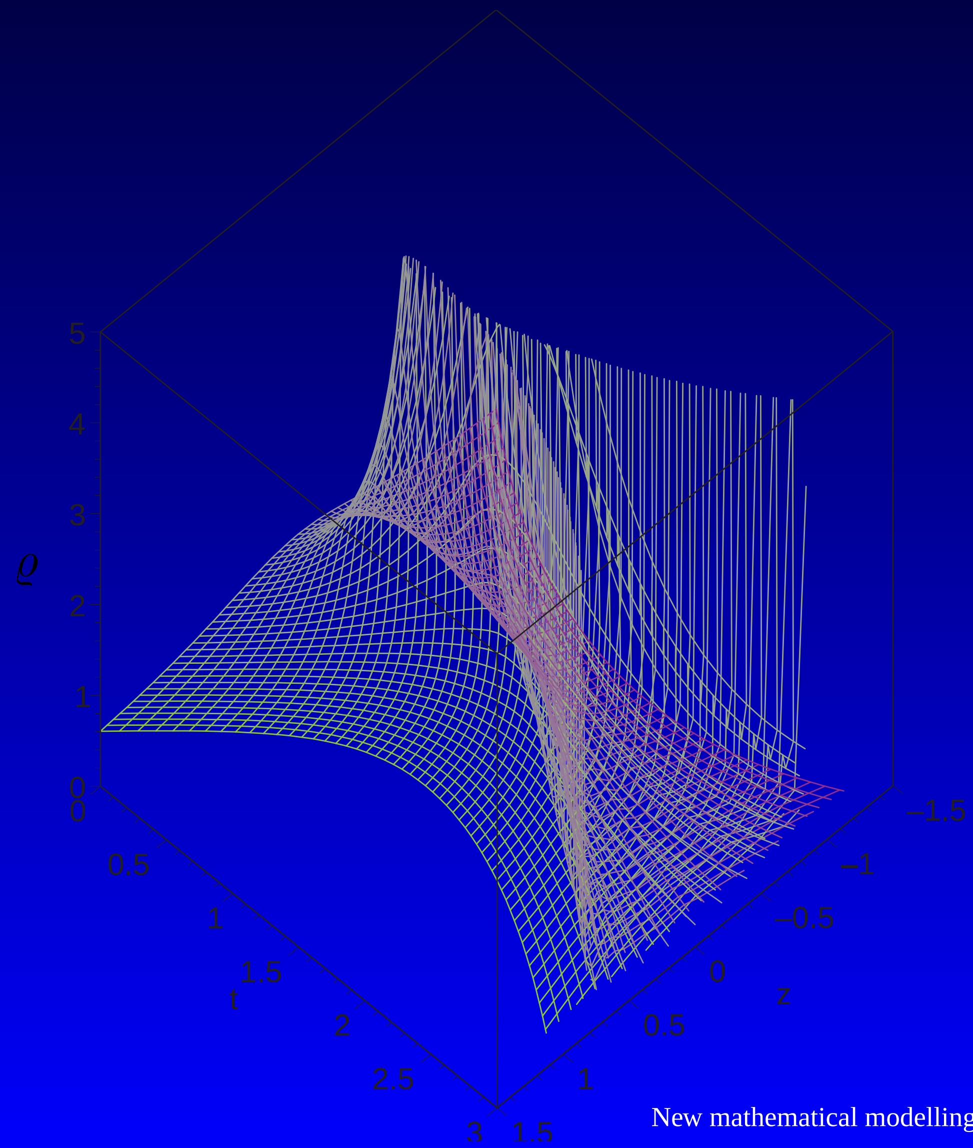
Bunch Constituent Worldlines

Colliding Charged Bunches Expanding Under Self Forces



Charge Density Singular Sheets

Dynamic formation of “multi-component”currents



Exact Symmetric Solutions

- Plane symmetric solutions reduce the system to a field theory on a 2-dimensional Lorentzian spacetime (with global coordinates t, z).
- The system is solved exactly using a co-moving coordinate system (τ, σ) adapted to the charged continuum.
- However, expressing the solutions in terms of laboratory coordinates (t, z) requires the inverse of the mapping $(\tau, \sigma) \longrightarrow (t, z)$, which is generally difficult to obtain in closed form.
- A running parameter $\varepsilon > 0$ is introduced into the mapping $(\tau, \sigma) \longrightarrow (t, z)$ and a perturbation scheme in ε facilitates an order-by-order construction of the inverse of the mapping $(\tau, \sigma) \longrightarrow (t, z)$ leading to 1-parameter families $(V^\varepsilon, \rho^\varepsilon, F^\varepsilon)$ of solutions in ε.
- $$F^\varepsilon = \Sigma_{n=-1}^{\infty} \varepsilon^n F_n, \quad V^\varepsilon = \Sigma_{n=-1}^{\infty} \varepsilon^n V_n, \quad \rho^\varepsilon = \Sigma_{n=1}^{\infty} \varepsilon^n \rho_n$$

  over some range of ε where the coefficients F_n, V_n and ρ_n are 2-forms, vector fields and scalar fields respectively.

Exact Solutions

Exact solutions take the form

$$F = \mathcal{E}(t,z)\, dt \wedge dz,$$

$$V = \frac{1}{\sqrt{1-\mu^2(t,z)}}\,(\partial_t + \mu(t,z)\partial_z)$$

where μ is the magnitude of the Newtonian velocity field of the charged continuum measured by an inertial (laboratory) observer.

The electric field satisfies

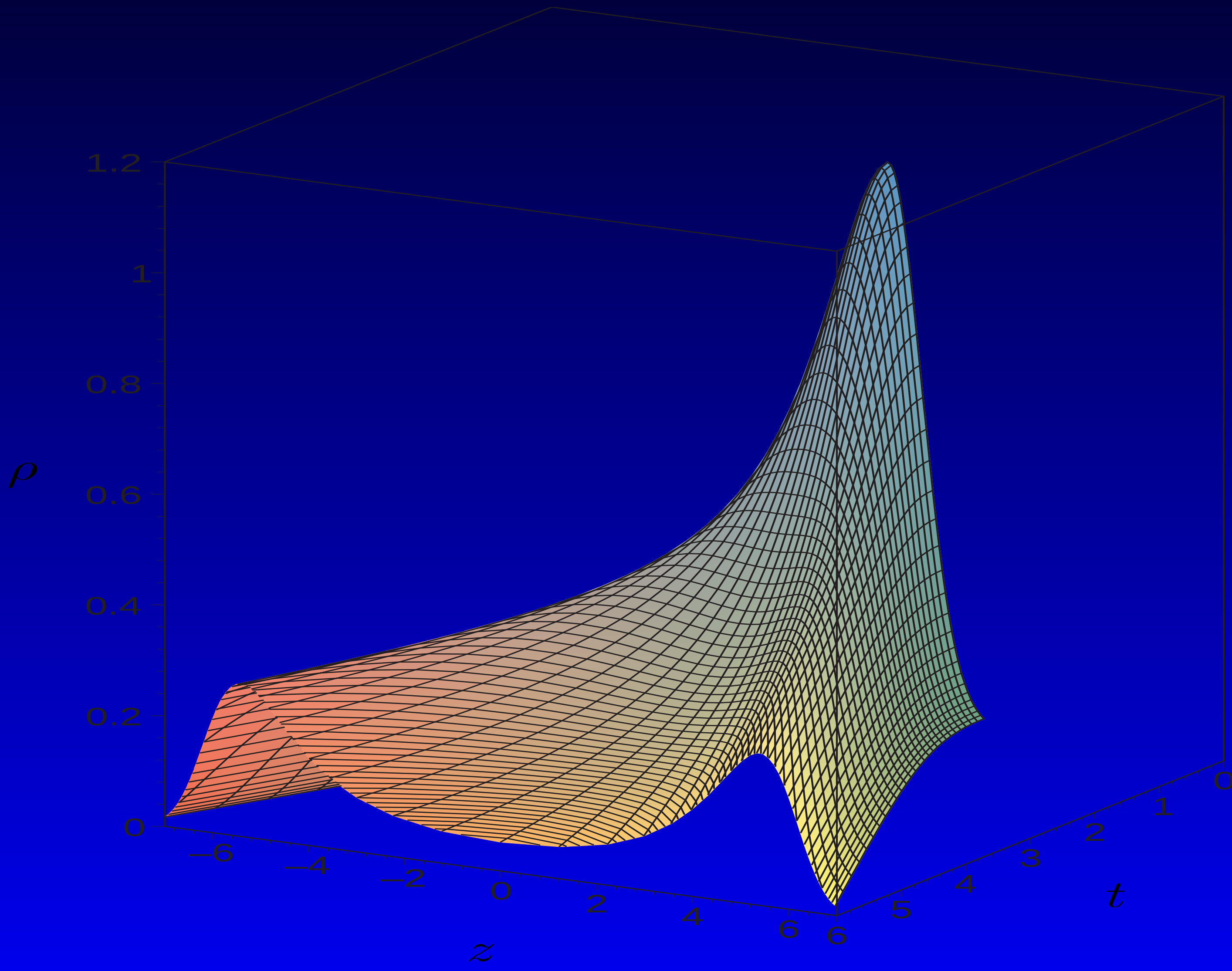
$$d\mathcal{E} = \rho\#\widetilde{V},$$

$$\nabla_V \widetilde{V} = \mathcal{E}\#\widetilde{V}$$

where $\#$ is the Hodge map associated with the volume 2-form $\#1 \equiv dt \wedge dz$ and $\mathcal{E}$ is constant along the integral curves of V.

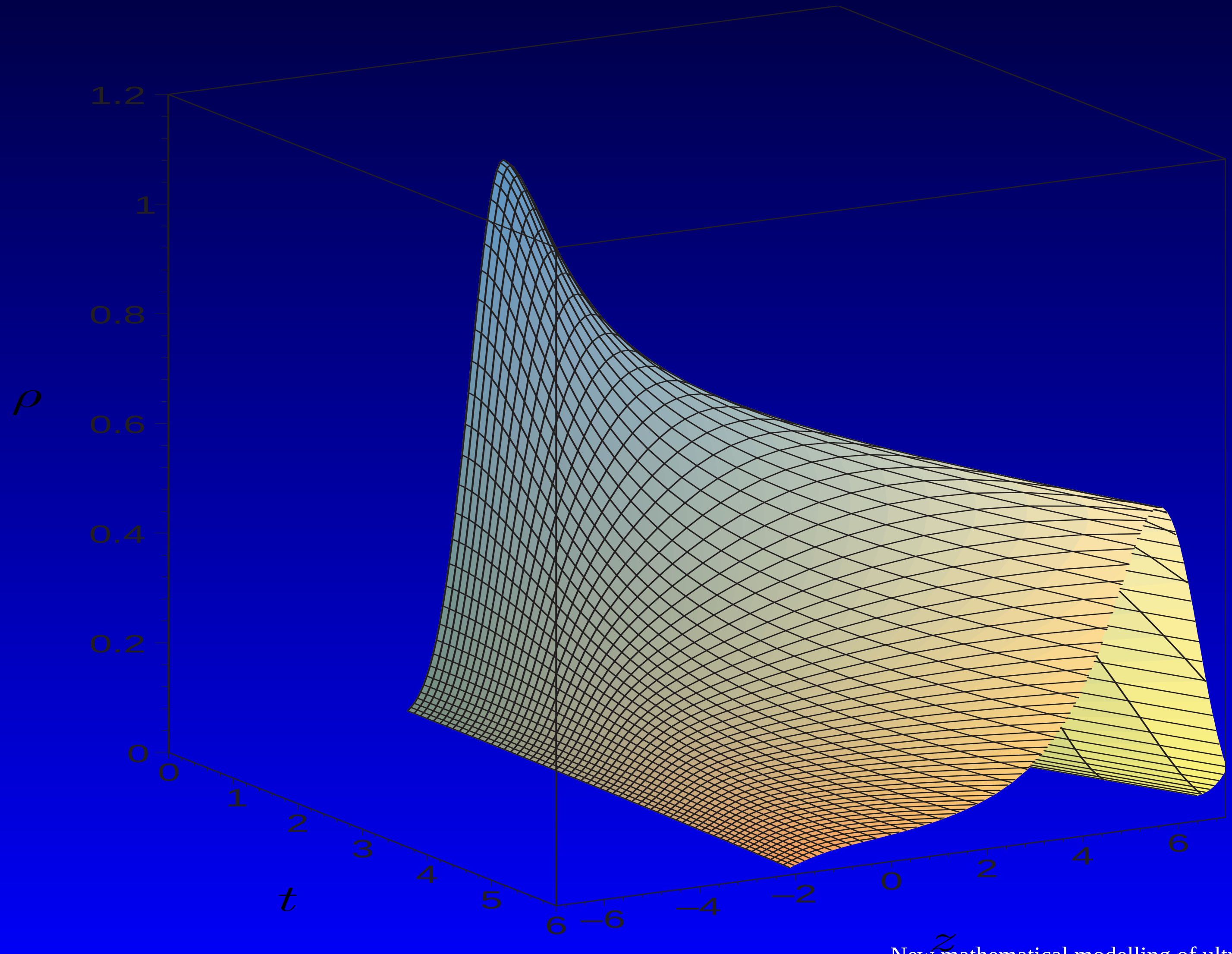
Charged Bunch

Charge Density ρ moving Under Self Field



Charged Bunch

Charge Density ρ moving Under Self and External Fields



Proposed New Representation

This example suggests that more general solutions be represented order-by-order in ε with.

$$F^\varepsilon = \Sigma_{n=-1}^{\infty} \varepsilon^n F_n, \qquad V^\varepsilon = \Sigma_{n=-1}^{\infty} \varepsilon^n V_n, \qquad \rho^\varepsilon = \Sigma_{n=1}^{\infty} \varepsilon^n \rho_n$$

where F_{-1} is an external field (a solution to the source-free Maxwell equations).

The electric 4-current then has the form

$$J^\varepsilon = \rho^\varepsilon V^\varepsilon = \Sigma_{n=0}^{\infty} \varepsilon^n J_n.$$

The above expansions partially decouple the non-linear field system yielding an infinite hierarchy of equations that are amenable to solution when supplemented with appropriate boundary conditions and initial data.

Perturbation about the Light Cone

- There exists a class of solutions representing configurations of charged particles in *ultra-relativistic* collective motion.
- To leading order the velocity field of the charged continuum is light-like.
- The full series will be considered as an asymptotic expansion for a solution and physically represents an ultra-relativistic configuration.
- Such configurations are chosen to be representative of the class relevant to charged beams in high-energy accelerators.

Conclusions

- A description based on charged continua rather than a collection of classical point particles has been explored.
- Pathologies (such as pre-acceleration) associated with radiating point particles are avoided by relying on field-theoretical notions.
- A novel analysis of charged beam dynamics has been presented and a model of a freely propagating charged bunch discussed. The approach relies on an asymptotic series representation of solutions to self-consistent spacetime covariant field equations for a charged continuum.

Conclusions

- The asymptotic series for the charge $4-$velocity field V is based on a light-like vector field (V_{-1}) that generates an ultra-relativistic approximation.
- The hierarchy of equations obtained are more amenable to analysis than the original non-linear field system and particular solutions have been presented.
- Avenues for development include ultra-relativistic charged beams in the vicinity of beam pipes, RF cavities, spoilers, etc. leading to dynamical effects that are often described in terms of “wake-fields” and a clearer understanding of radiation-reaction and coherent radiation exhibited by continuum models of charged particle beams.

References

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Multiple currents in charged beams, EPAC 06:

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