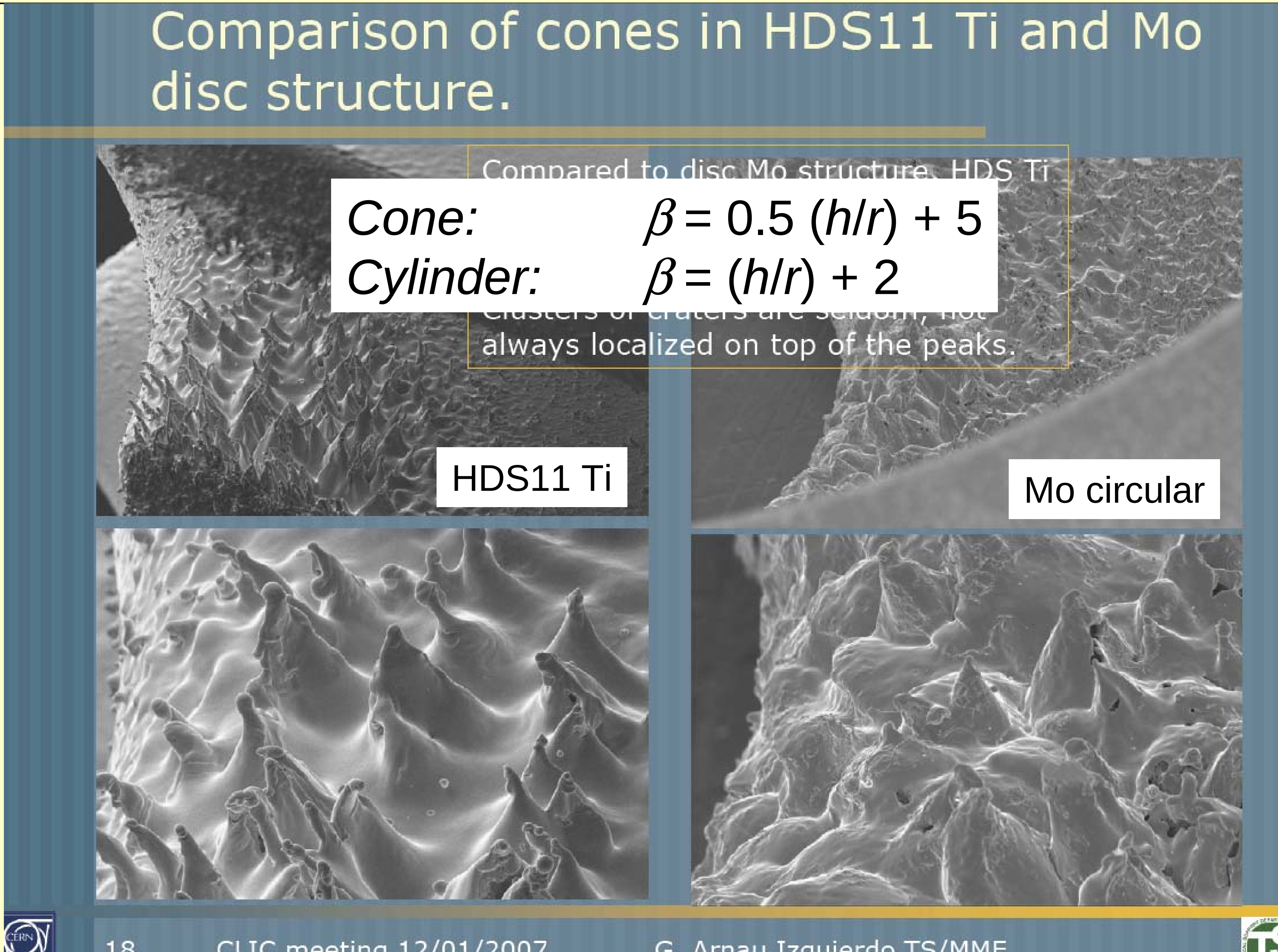
A few considerations on breakdown phenomena

- Some "anecdotes"
  - Beta values from SEM
  - Taylor cones
- Temperature rise calculations
  - 1D, 2D, 3D heating
  - Heating of tips by field emission currents
- (Nervous…) breakdown rate

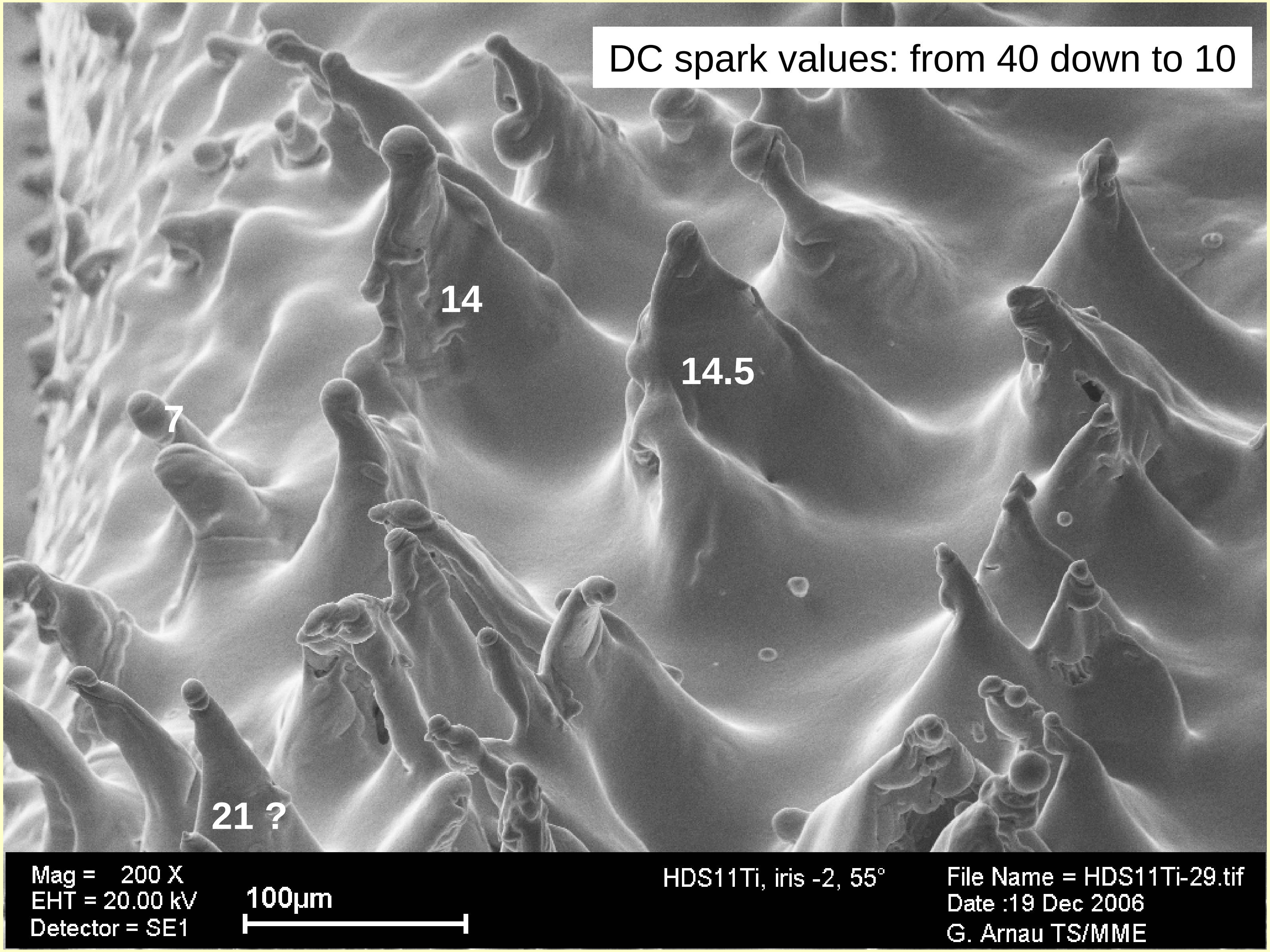
From Gonzalo Arnau Izquierdo



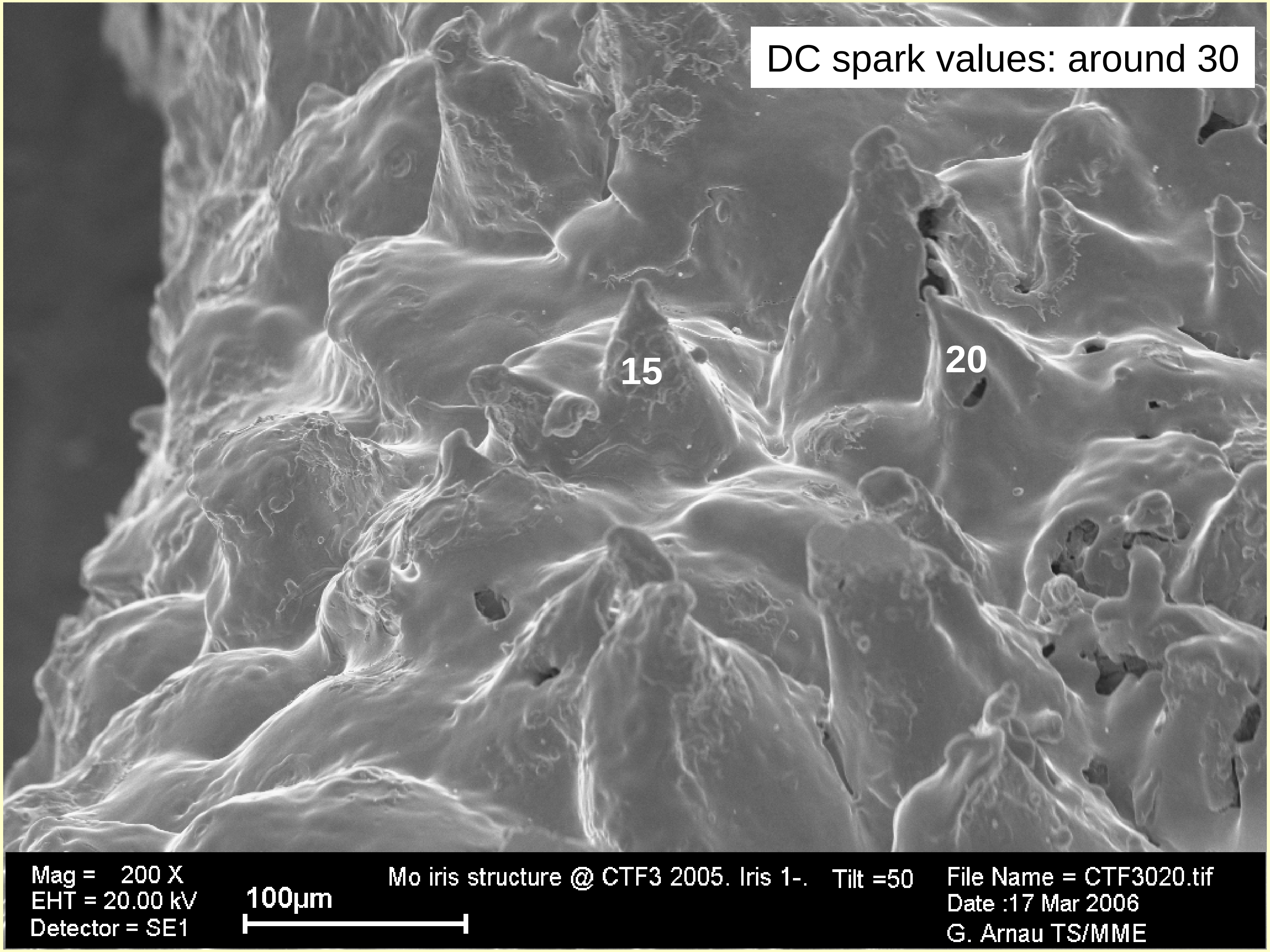
Cone: $\beta = 0.5\,(h/r) + 5$

Cylinder: $\beta = (h/r) + 2$

Beta calculations from SEM observation - Ti



Beta calculations from SEM observation - Mo

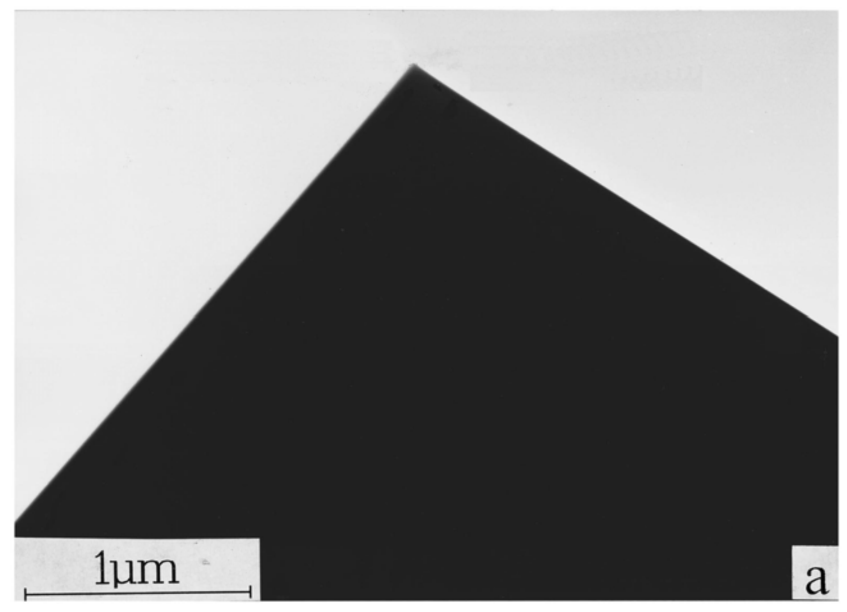


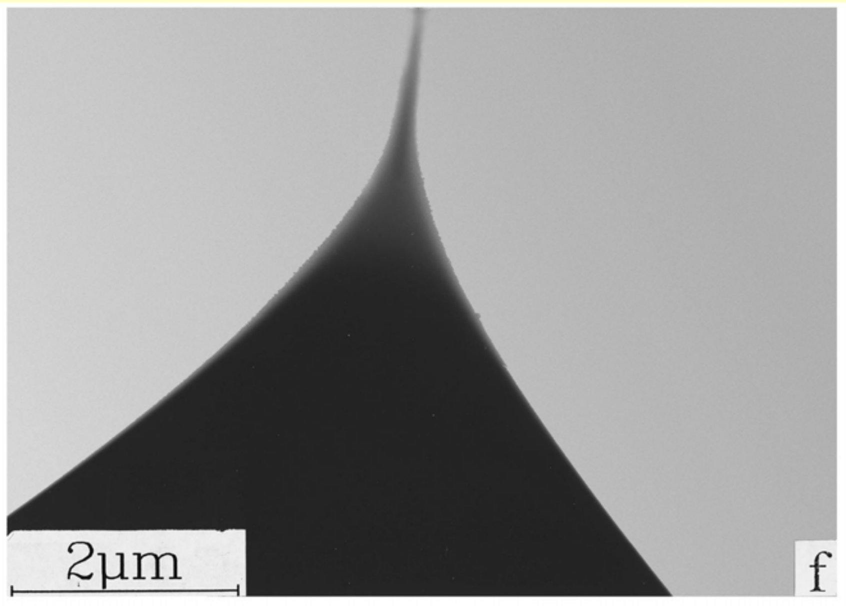
Pulling of liquid? Taylor cones

- The “cones” might be the result of the E field pulling over the molten metal. Models of this process exist in the literature.

- When a molten metal is pulled with an electric field, the metal surface is deformed. The resulting shape is due to the balance between the electrostatic force and the surface tension.
- At the highest field the limiting shape is conical, with an half-opening angle of 49.3 degrees (Taylor cone, Proc. Roy Soc. A 280 (1964) 383).
- This shape is independent of the material. When further increasing the field, ion emission starts with a jet. Locally, the atoms binding energy is overcome by electrostatic forces.
- The shape and dynamics of the jet depend on viscous forces
- This process is used in so-called Liquid Metal Ion Sources (for example Cs)

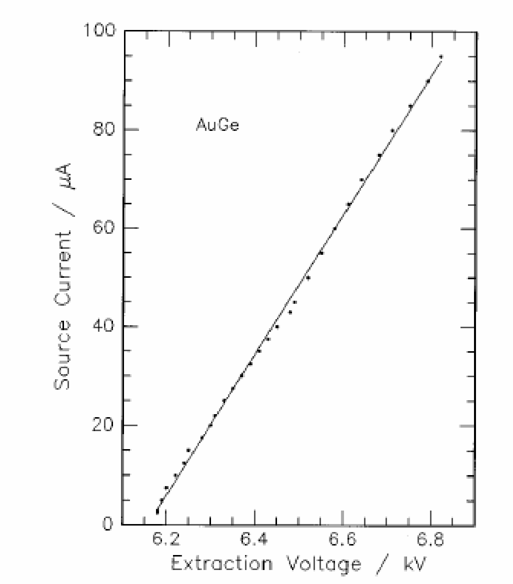
Taylor cones - images





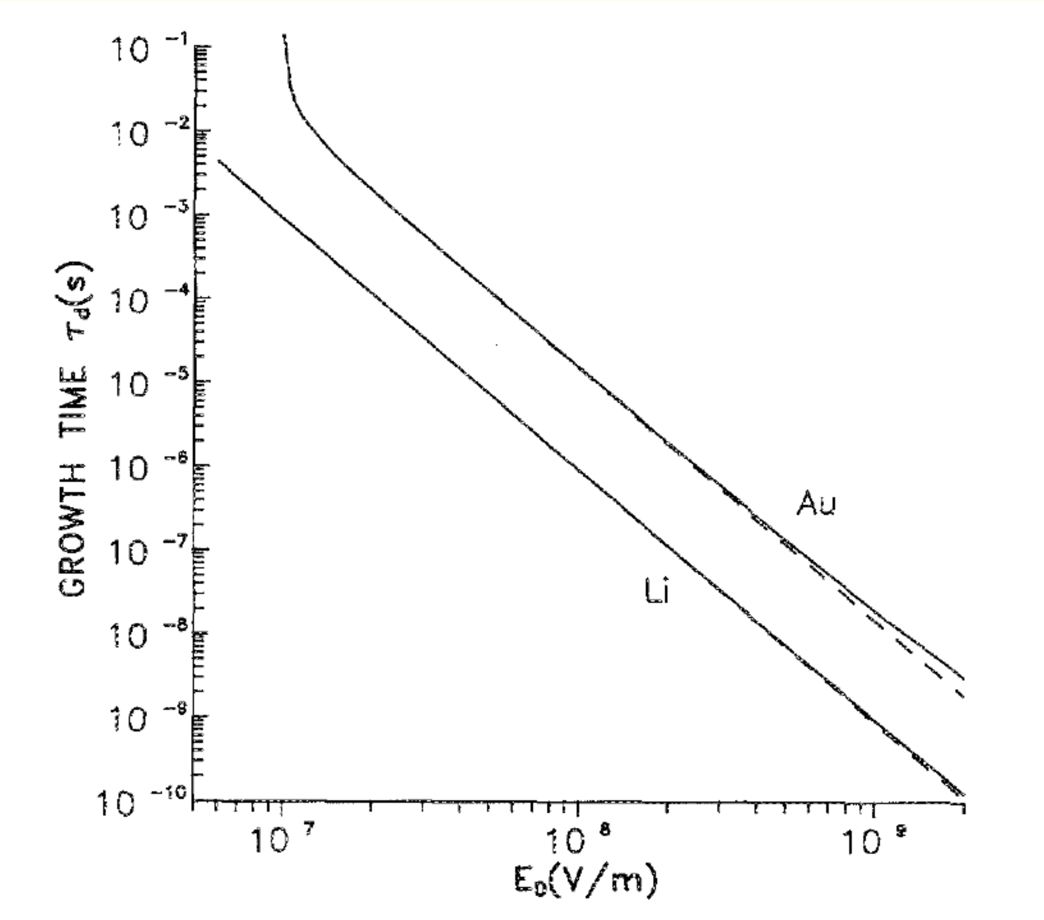
(From Driesel et al.J. Vac. Sci. Technol. B 14 (1996) 3367) - AuGe alloy

- Shape with minimal ion emission (angle close to theoretical value) – left
- Shape with strong ion jet emission - right. Ion current 95 μA, Field 6.8 kV / 1.5 mm, estimated jet diameter 175 nm.
- $\Rightarrow$ flux = 6 * 10^{14} ions/sec
- $\Rightarrow$ equivalent pressure = 2.5 bar



Taylor cones: time for formation I

- Formation of instability waves on a flat molten surface (He et al. J. Appl. Phys. 68 (1990) 1475



- Seems slow compared to CLIC situation

Taylor cones: time for formation II

- Growth time of cones (Suvarov et al. J. Appl. Phys. D 33 (2000) 1245) – Mercury at 240 MV/m applied field

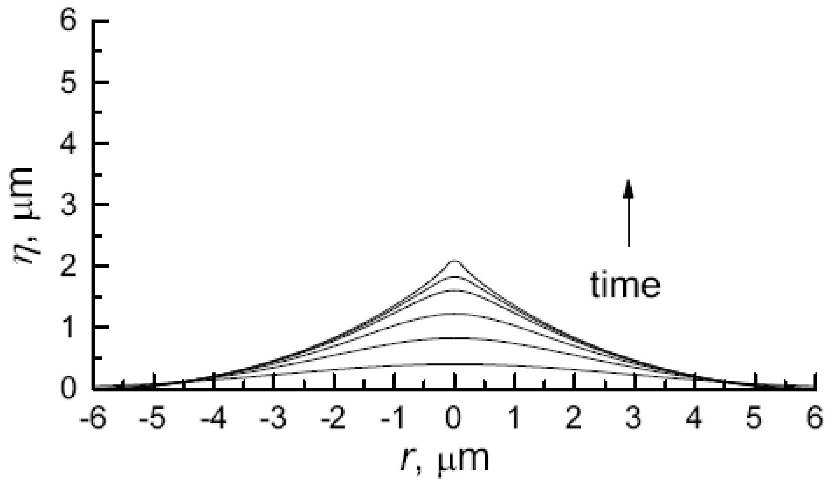


Figure 2. The surface time evolution; initial form is Gaussian with $\lambda = 4$ μm, $h = \lambda/10$. The surfaces are consecutively represented at time: 0, 0.56, 0.73, 0.81, 0.83 and 0.85 μs.

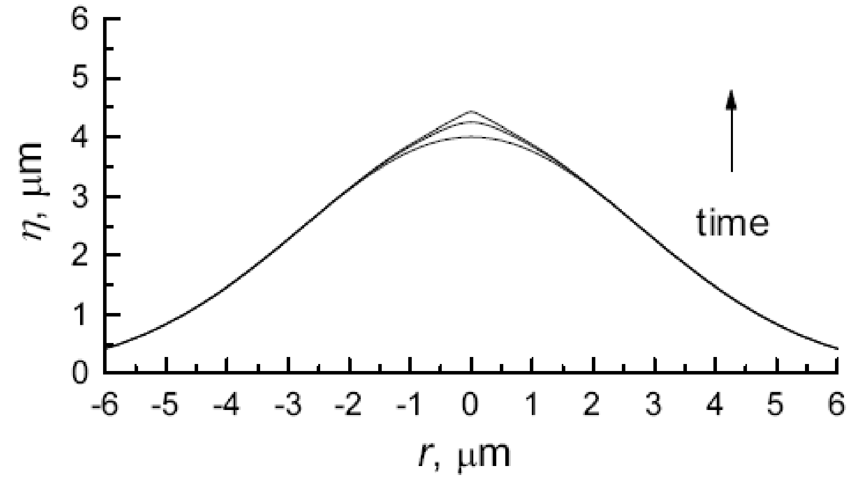


Figure 3. The surface time evolution; initial form is Gaussian with $\lambda = 4$ μm, $h = \lambda$. The surfaces are consecutively represented at time: 0, 0.048 and 0.057 μs.

- Growth time can be very fast depending on starting conditions

Temperature rise calculations

- Here starts the main part of the talk

- 1D, 2D, 3D time dependent heating
  - Relevant for the discussion on breakdown limit

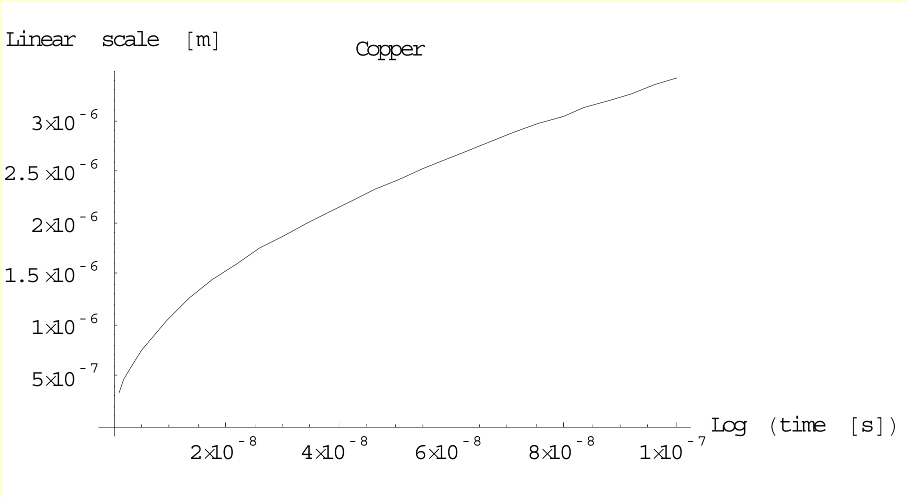
- Heating of tips by field emission currents
  - Relevant for the discussion on breakdown probability

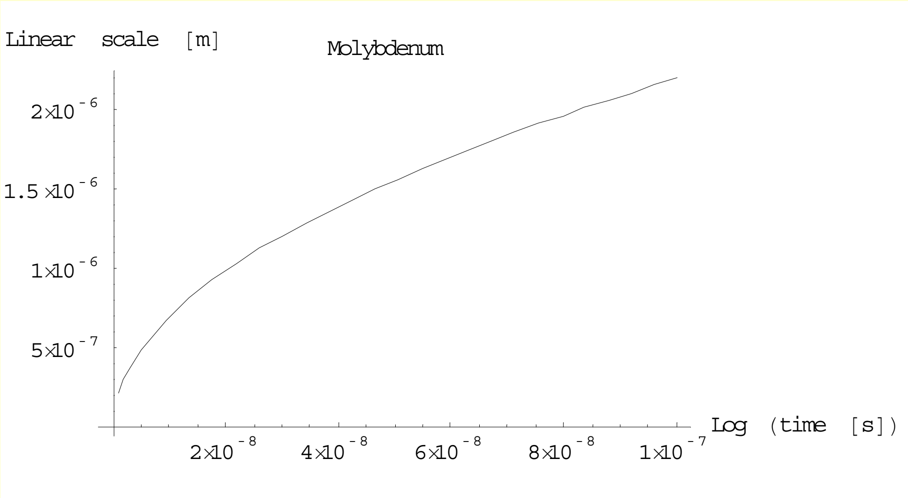
Time-dependent heating

- The breakdown limit of materials in RF tests is observed to follow the dependence: $P\tau^a$ with a=1/3 for copper and a=2/3 for molybdenum
- Is there any intrinsic material dependence? Heat flow equation:

$$\nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

- With: k = thermal conductivity, $\alpha = k/(c^*\rho)$, c = specific heat, ρ = density
- In-time dependent calculations the distinction between a “fast” and “slow” regime is based on the diffusivity time $\tau_D = R^2/\alpha$. R is the linear scale of the phenomena that are under consideration

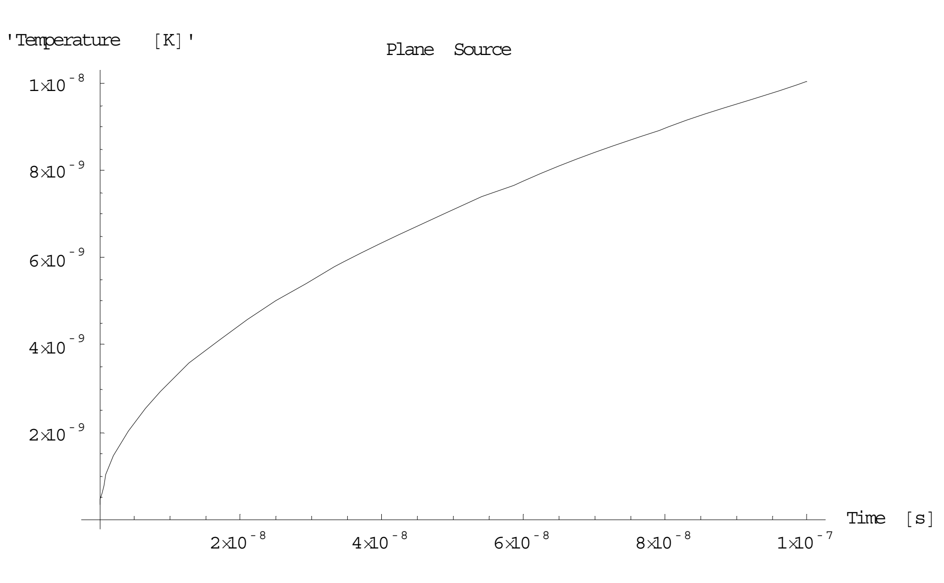


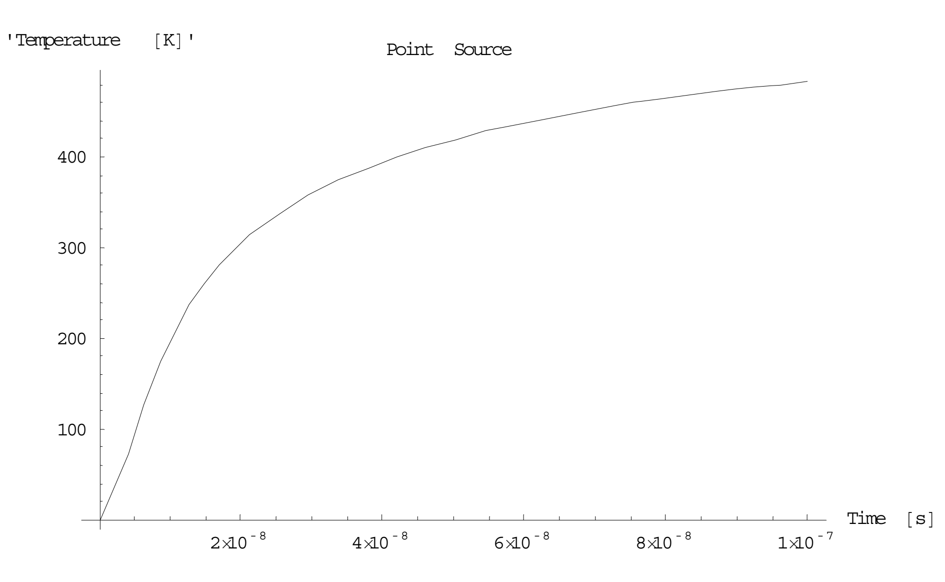


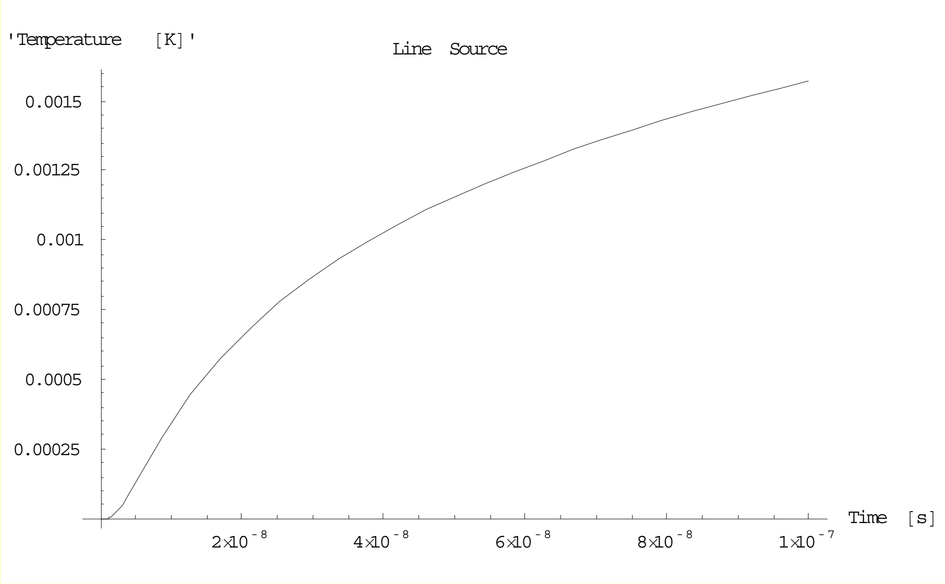
1D, 2D, 3D heating profiles inside a solid, or over a semi-infinite solid

- Clockwise:
- 1D heat flow → plane source gives square-root time dependence

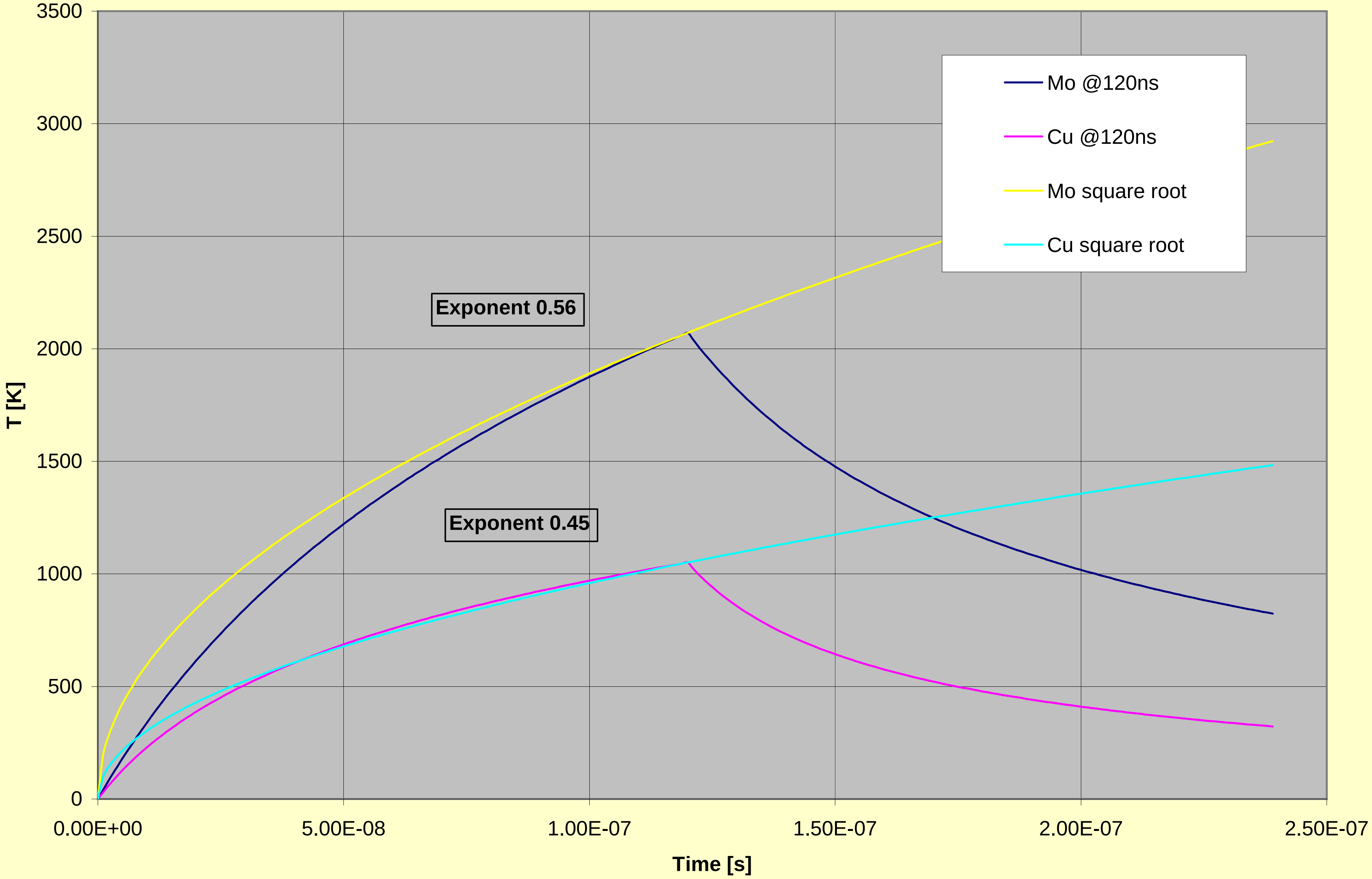
- 2D heat flow → line source
- 3D heat flow → point source







From Alessandro Bertarelli: 2µm x 2µm heat source

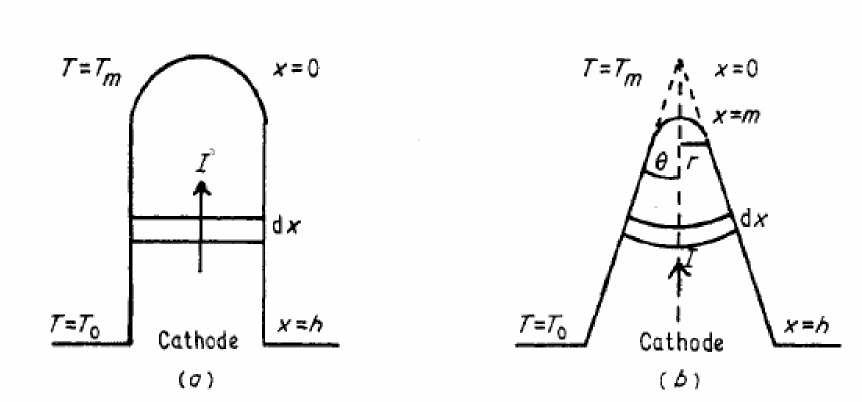


Heating by field emission

- Field emission currents heat a (conical) tip by Joule effect. The tip is assumed to have a fixed temperature at its base and have a temperature gradient along its height.
- If the resistivity is considered temperature-independent, a stable temperature is achieved (Chatterton Proc. Roy. Soc. 88 (1966) 231
- If the resistivity (and the other material parameters to a lesser extent) is temperature dependent, then when it increases there is a larger power dissipation, resulting in a further increase in temperature and so on (Williams & Williams J. Appl. Phys. D 5 (1972) 280).
- Below a certain current threshold, a stable regime is reached
- Above the threshold, a runaway regime is demonstrated

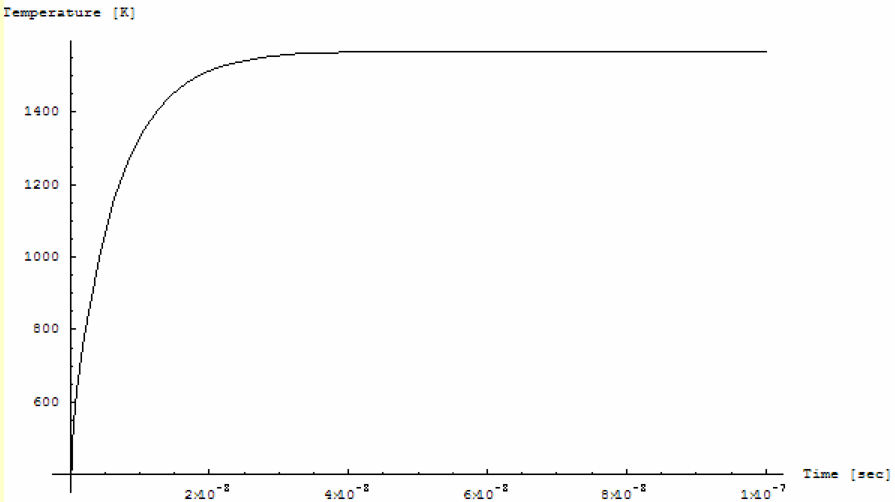
- The T(t) can be calculated.

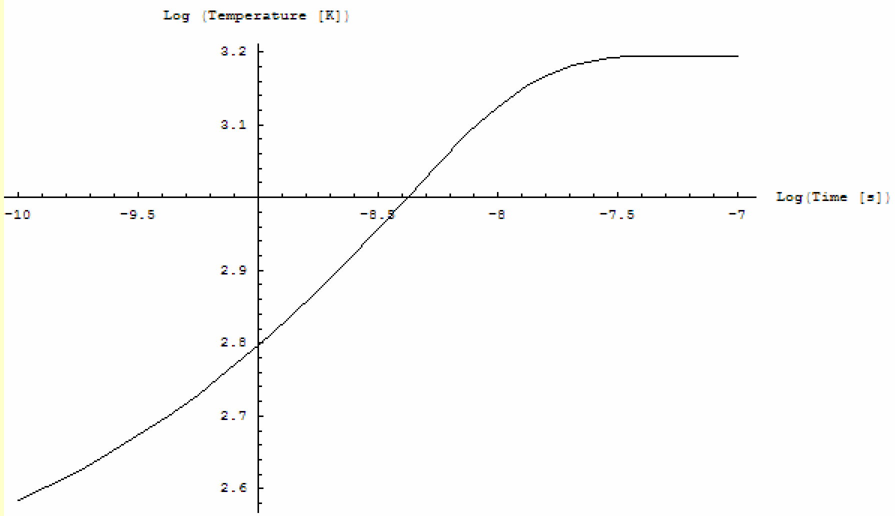
$$\frac{\partial T}{\partial t}=\frac{K}{\sigma S}\left(\frac{\partial^2 T}{\partial x^2}+\frac{2}{x}\frac{\partial T}{\partial x}\right)+\frac{\rho}{\sigma S}\left(\frac{I}{2\pi x^2(1-\cos\theta)}\right)^2.$$



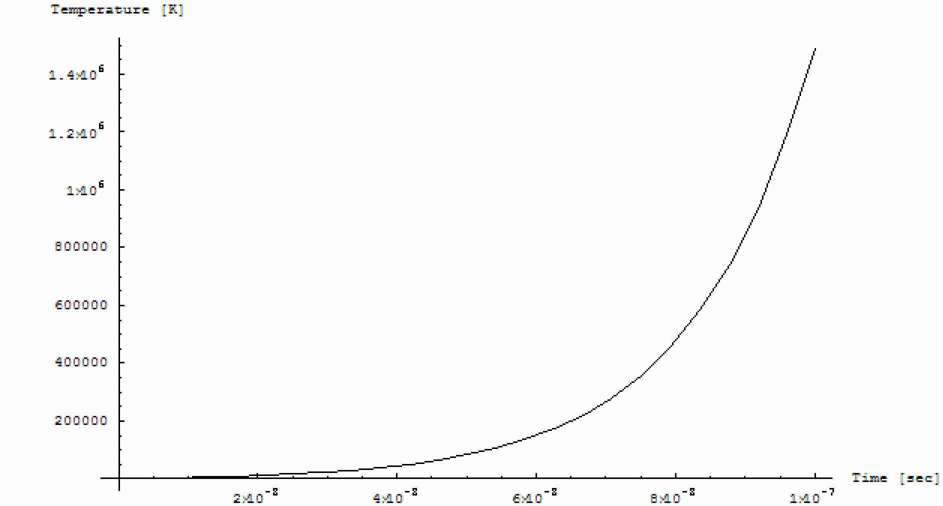
Simulation for Mo cone: diameter 20 nm, beta = 30

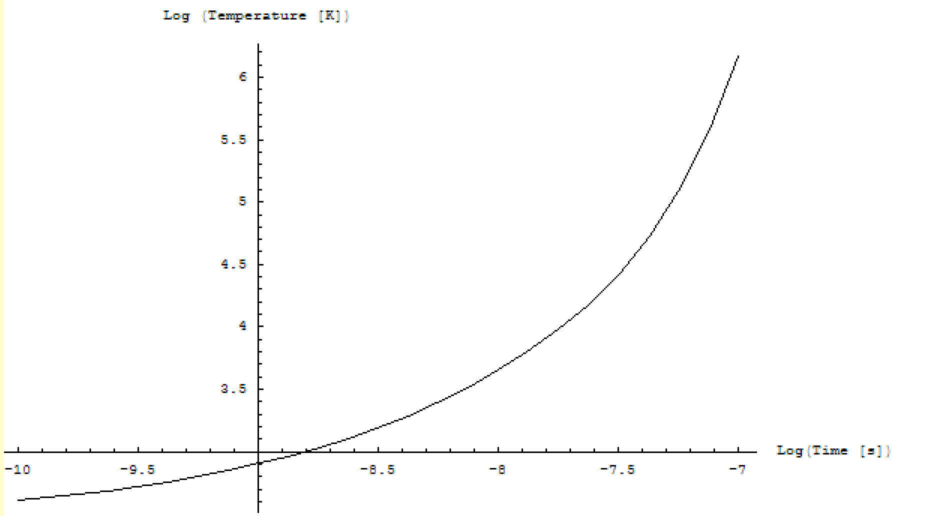
E=374 MV/m





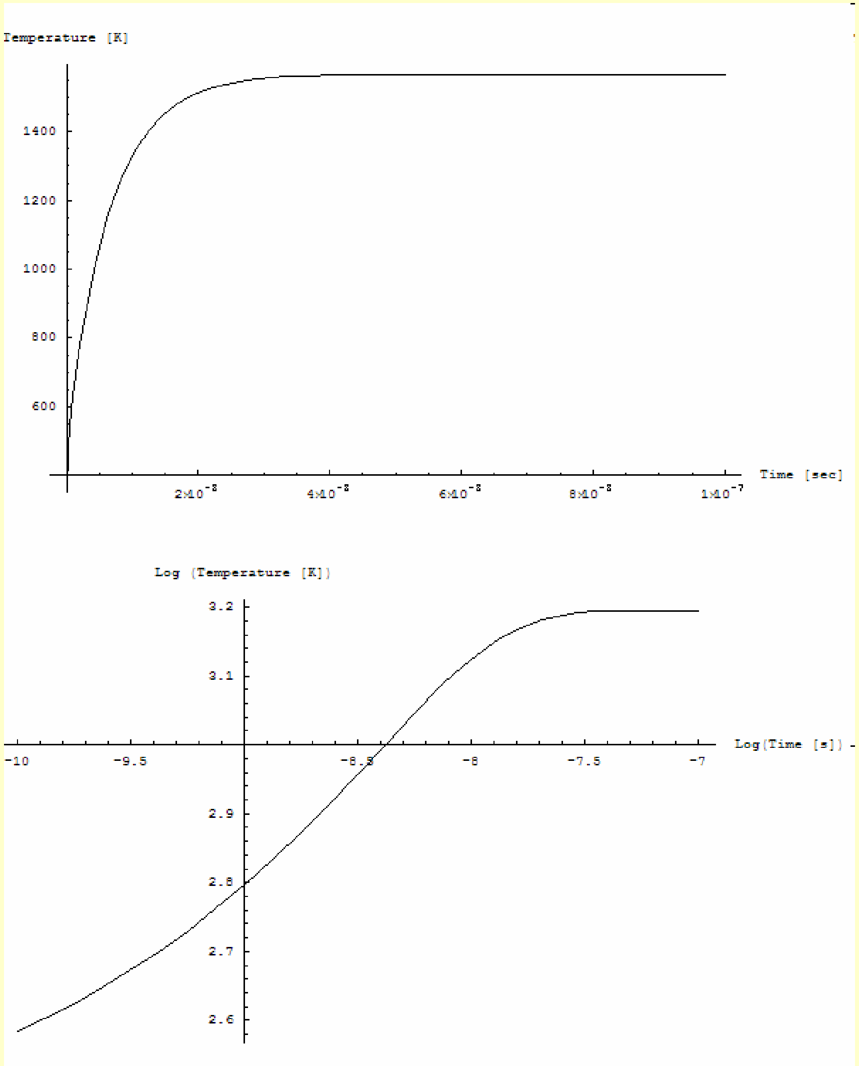
E=378 MV/m



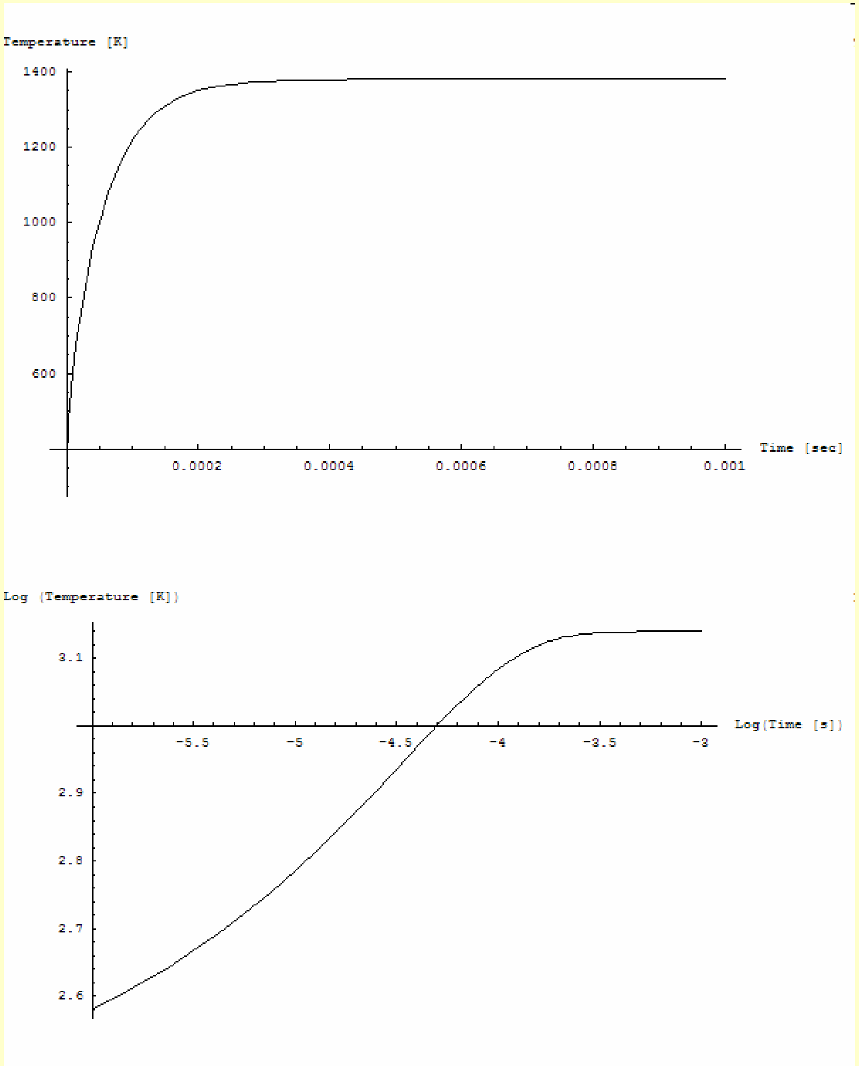


Simulation for Mo cone: beta = 30

Diameter 20 nm, E=374 MV/m, current = 0.028 A



Diameter 2000 nm, E=226 MV/m, current = 2.8 A



Heating by field emission II

- The current threshold for runaway depends on the diameter of the cone
- The time constant appears to depend on the (diameter)^2 of the cone
- The final temperature (if stable) depends on (thermal conductivity)$^{-2}$
- The rate of temperature increase is (time)$^{0.5}$ below runaway
- The rate of temperature increase depends on (thermal conductivity)$^{-0.5}$ (???)
- The dependences on the electrical conductivity are the same as for the thermal conductivity

Thoughts on breakdown rate

- Is it possible to model the breakdown rate probability starting from simple phenomena?

Comparison with breakdown rate measurements?

- The breakdown probability: $P(x_1, x_2, ..., x_n) = f(x_1) f(x_2) ... f(x_n)$
- Where x_i might be E, τ or a even a combination of these or other physical quantities.
- I make the assumption that the ignition of a breakdown is due only to gas ionisation by electrons. A breakdown is of course an ionisation cascade
- I assume that the probability of igniting a cascade depends linearly on the amount of gas available and on the primary electron current
- In this case:

$$P_{breakdown} \propto I_{electrons}\, pressure_{gas}$$

- Normalisation should of course be applied
- Where do the electrons and the gas come from?

Comparison with breakdown rate measurements?

- The electron current is given by the standard Fowler-Nordheim equation:

$$I_{electrons} = FN(\beta E)$$
$$FN(\beta E) = Const * (\beta E)^2 \exp(-B/\beta E)$$

- The constant includes the emitter area

- The gas molecules that get ionised (and allow me this far-fetched assumption!) are indeed the metal vapours created at the tip of the emitters, because of Joule heating by the F-N current.
- It is very difficult to use the full heating model seen before. I made the very crude assumption that the temperature grows with $(\text{time})^{0.5}$ and scales inversely with the $(\text{thermal conductivity})^{0.5}$.
- The vapour pressure is then given by:

$$p = p_0 \exp\left(\frac{-H_0}{RT}\right)$$

- Where H_0 is the heat of vaporisation and R the gas constant. p_0 is a normalisation factor, there is a ratio of approximately 10^2.5 between Mo and Cu

Comparison with breakdown rate measurements?

- All this gives (k is the thermal conductivity, τ the length of the RF pulse):

$$P_{breakdown} \propto I_{FN}(\beta E) * p_0 \exp\left(\frac{-H_0 k^{0.5}}{C\tau^{0.5} J_{FN}^2}\right)$$

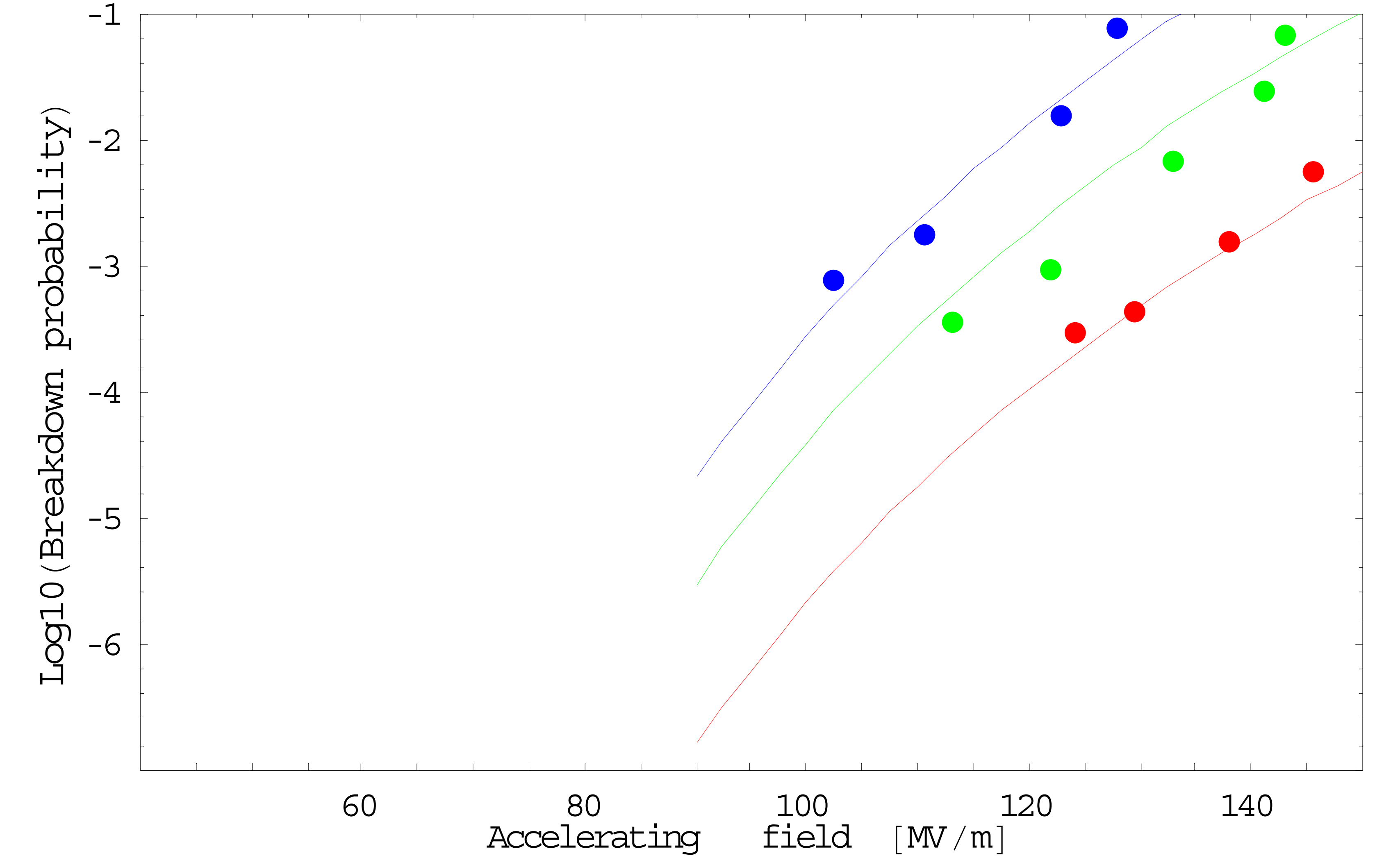
- Taking the Log_{10}, and applying a single proportionality constant for all the multiplicative factors (only the exponential part of the F-N equation is used):

$$Log(P_{breakdown}) = A + p_0 + 2Log(\beta E) - \frac{B}{\beta E} - \frac{H_0 k^{0.5}}{C\tau^{0.5} J_{FN}^2}$$

- Where *A, B, C* are fit to the experimental data (and include for example the ionisation cross section, the field emitter area, the probability normalization…)

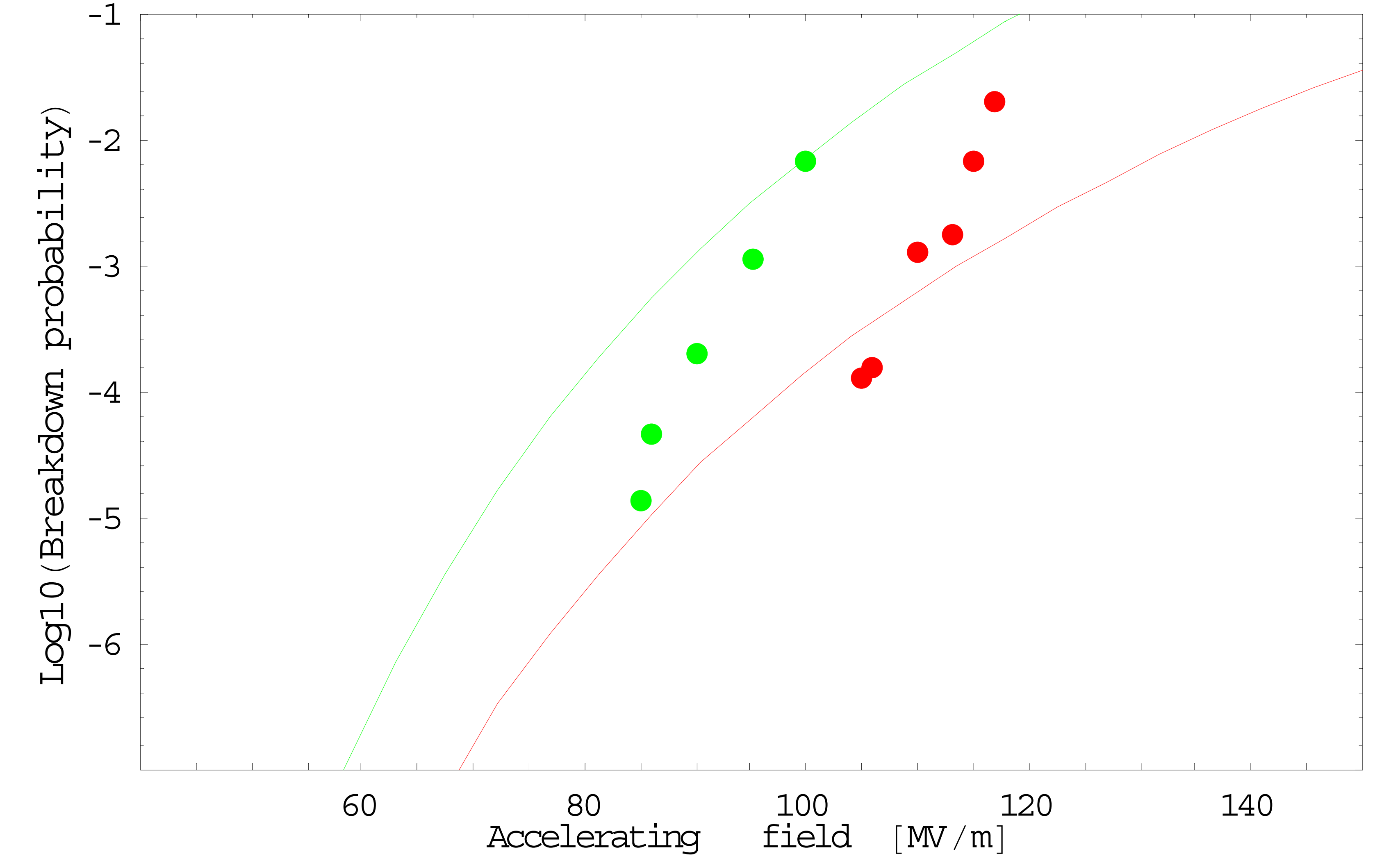
Fit to Mo data, 30 GHz circular iris

- $\beta = 30$, k = 138 $Wm^{-1}K^{-1}$, $p_0 = 10$^14.5 mbar, H_0 = 598 kJ/mol



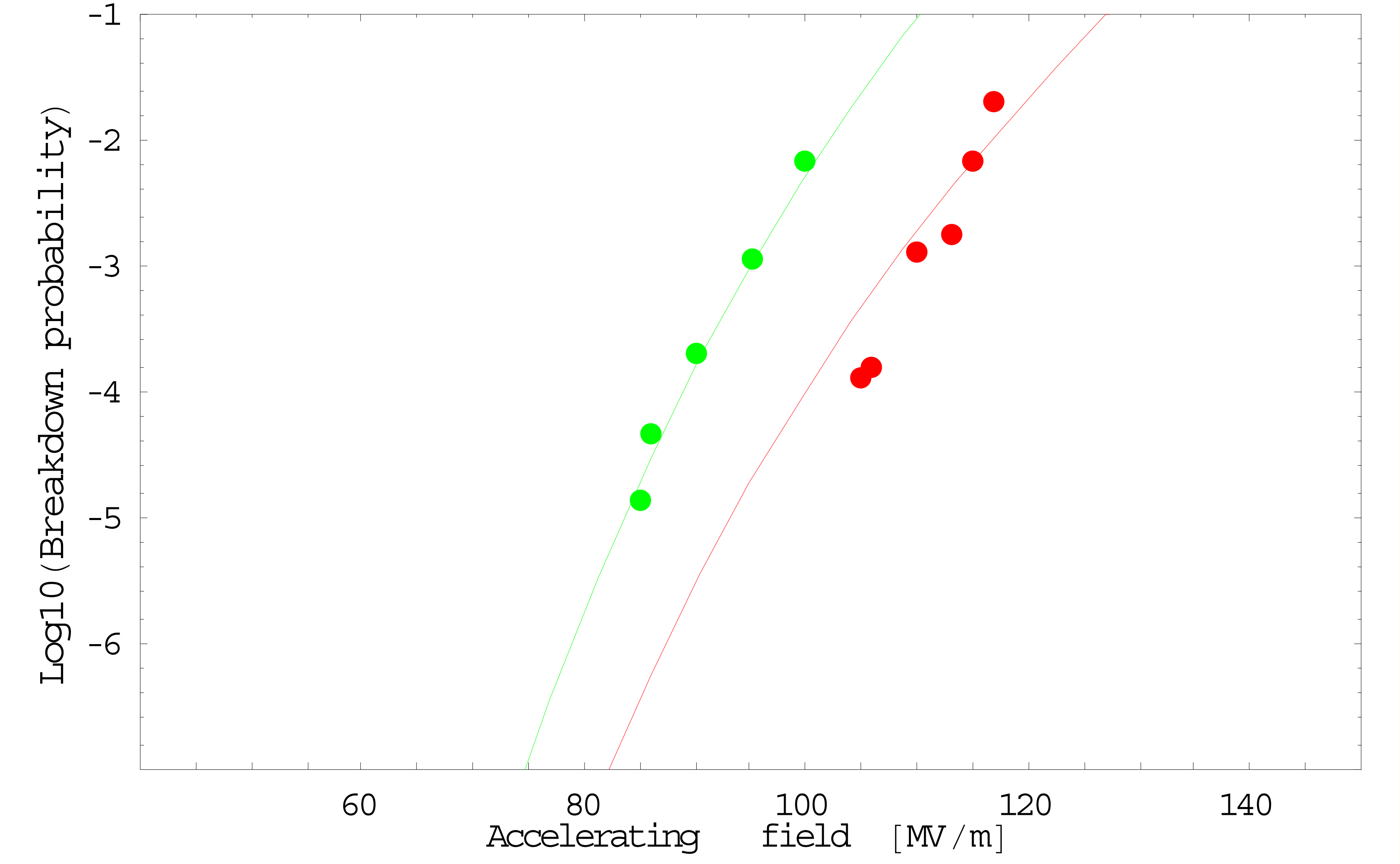
Keeping the same fit parameters and comparing to Cu data, 30 GHz

- $\beta = 45$, $k = 400$ Wm^{-1}K^{-1}, $p_0 = 10$^^{12} mbar, $H_0 = 300$ kJ/mol.



Letting free the F-N fit parameters and comparing to Cu data, 30 GHz

- *B* doubles and *A* increases of 6 units



Conclusions